MATHEMATICAL METHODS
(MTH2001)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The function f(z) is not differentiable at $z = z_0$ then z_0 is called
(a) an ordinary point of f(z) (b) a singular point of f(z)
(c) zero of f(z) (d) a non-singular point of f(z)

(ii) For an infinite power series $\sum_{n=0}^{\infty} c_n x^n$ if $\lim_{n\to\infty}\left|\frac{c_m}{c_{m+1}}\right| = R$ then the radius of convergence is
(a) 1/R (b) R (c) 1 (d) ∞

(iii) What is the general form of a power series centred at $x = c$?
(a) $\sum_{n=1}^{\infty} a_n (x-c)^n$ (b) $\sum_{n=1}^{\infty} a_n (x-c)$
(c) $\sum_{n=1}^{\infty} a_n x^n$ (d) $\sum_{n=1}^{\infty} a_n (x+c)^n$

(iv) What is the value of the second Legendre polynomial $P_2(x)$?
(a) $\frac{1}{2}(3x^2-1)$ (b) $2x^2-1$ (c) $\frac{1}{2}(5x^2-3)$ (d) $3x^2-1$.

(v) What is the standard form of Bessel's differential equation?
(a) $x^2y'' + xy' + (x^2-n^2)y = 0$ (b) $y'' + xy' + (x^2-n^2)y = 0$
(c) $x^2y'' + 2xy' + (x^2-n)y = 0$ (d) $y'' + y' + xy = 0$

(vi) Which method is typically used to solve linear PDEs by separating variables?
(a) Method of characteristics (b) Fourier transform
(c) Separation of variables (d) Laplace transform.

(vii) In the classification of second-order PDEs, the equation $u_{xx} + 2u_{xy} + u_{yy} = 0$ is
(a) Elliptic (b) Parabolic
(c) Hyperbolic (d) Can not be classified.

(viii) Lagrange's method is based on solving the characteristic equations of the form
(a) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ (b) $\frac{dx}{P} + \frac{dy}{Q} = \frac{dz}{R}$
(c) $Pdx + Qdy + Rdz = 0$ (d) $Pdx + Qdy = Rdz$

(ix) Which of the following is a second-order partial differential equation?

(a) $u_x + u_y = 0$ (b) $u_{xx} + u_{yy} = 0$

(c) $u_t + u_x = 0$ (d) $u_t - u_x = 0$.

(x) Complementary function of the given PDE $(D^2 - 7DD' + 6D'^2)z = 0$ is given by:

(a) $\phi_1(y+x) + \phi_2(y-6x)$ (b) $\phi_1(y+x) + \phi_2(y+6x)$

(c) $\phi_1(y+x).\phi_2(y+6x)$ (d) $\phi_1(y-x) + \phi_2(y-6x)$.

Fill in the blanks with the correct word

(xi) If the difference between the roots of the indicial equation is an integer, the second solution may involve a ____________________ term.

(xii) The generating function for Hermite polynomials $H_n(x)$ is ________________.

(xiii) The general solution in Lagrange's method is expressed as a function of the two independent integrals, $f(u,v) = 0$, where u and v are the ___________.

(xiv) The method of separation of variables assumes that the solution of a partial differential equation can be written as a product of _____ functions, each depending on a single independent variable.

(xv) The Laplace transform is useful for solving boundary value problems because it converts partial differential equations into _____ differential equations.

Group - B

2. (a) Find power series solution of the following differential equation $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - xy = 0$ about the point $x = 0$. *[(MTH2001.1,MTH2001.2)(Evaluate/HOCQ)]*

(b) Determine the radius of convergence for the following power series $\sum_{n=1}^{\infty} \frac{(-3)^n}{n \times 7^{n+1}}(x-5)^n$ *[(MTH2001.1,MTH2001.2)(Apply/IOCQ)]*

8 + 4 = 12

3. (a) Find the series solution of the equation $y'' - xy' + x^2y = 0$ about $x = 0$. *[(MATH2001.1, MATH2001.2) (Apply/IOCQ)]*

(b) Using Frobenius method solve the differential equation $\frac{d^2y}{dx^2} + \left(\frac{1}{4x}\right)\frac{dy}{dx} + \left(\frac{1}{8x^2}\right)y = 0$. *[(MTH2001.1, MTH2001.2) (Remember/LOCQ)]*

7 + 5 = 12

Group - C

4. (a) State the generating function of the Bessel's function $J_n(x)$. Use this to prove that $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$. *[(MTH2001.3)(Remember/LOCQ)]*

(b) Prove that $P_n(1) = 1$, where $P_n(x)$ is a Legendre polynomial. *[(MTH2001.3)(Analyse/IOCQ)]*

7 + 5 = 12

5. (a) Prove that $\lim_{z \to 0} \frac{J_n(z)}{z^n} = \frac{1}{2^n \Gamma(n+1)^n}$, where $n > -1$. *[(MTH2001.3)(Evaluate/HOCQ)]*

(b) Express $x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Hermite's polynomials.

[(MTH2001.3)(Evaluate/HOCQ)]

6 + 6 = 12

Group - D

6. (a) Form a partial differential equation by eliminating the arbitrary function ϕ from the relation $\phi(x + y + z, x^2 + y^2 + z^2) = 0$. *[(MTH2001.4, MTH2001.5)(Remember/LOCQ)]*

(b) Solve $p + 3q = 5z + \tan(y - 3x)$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

[(MTH2001.4,MTH2001.5)(Apply/IOCQ)]

6 + 6 = 12

7. (a) Solve $(1 + y)p + (1 + x)q = z$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

[(MTH2001.4,MTH2001.5)(Apply/IOCQ)]

(b) Find the differential equation of all spheres of radius c, having centres on xy-plane. *[(MTH2001.4, MTH2001.5)(Apply/IOCQ)]*

6 + 6 = 12

Group - E

8. (a) Solve the boundary value problem $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ given $u(0,t) = 0, u(l, t) = 0$, $u(x,0) = f(x)$ *and* $\frac{\partial u}{\partial t}|_{(x,0)} = 0$ when $0 < x < l$, where, l is the length of the string. *[(MTH2001.5,MTH2001.6)(Evaluate/HOCQ)]*

(b) Solve: $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \sin(x + y)$. *[(MTH2001.5,MTH2001.6)(Remember/LOCQ)]*

8 + 4 = 12

9. (a) Applying the method of separation of variables, solve the BVP $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ if $u(0, y) = 8e^{-3y}$ (where u is a function of x and y).

[(MTH2001.5, MTH2001.6) (Apply/IOCQ)]

(b) Solve: $(D^2 - DD' - 2D'^2)z = (y - 1)e^x$, where $D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}$.

[(MTH2001.5,MTH2001.6)(Remember/LOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	35.42	35.42	29.16