ENGINEERING COMPUTATIONAL TECHNIQUES
(MECH 4124)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) If X is the true value of a quantity and X_1 is its approximate value, then the relative error is
(a) $\frac{|X_1-X|}{X_1}$ (b) $\frac{|X-X_1|}{X}$ (c) $|X_1/X|$ (d) $X/|X_1-X|$

(ii) The error in the trapezoidal rule for numerical integration is
(a) $O(h)$ (b) $O(h^2)$ (c) $O(h^3)$ (d) $O(h^4)$

(iii) If $f(x)=0$ is an algebraic equation, the Newton-Raphson method for approximating the root of the equation is given by $x_{n+1}=x_n-f(x_n)/?$
(a) $f(x_{n-1})$ (b) $f'(x_{n-1})$ (c) $f'(x_n)$ (d) $f''(x_n)$

(iv) In LU decomposition method of solution the coefficient matrix is decomposed into
(a) Lower and upper triangular matrix both
(b) Lower triangular matrix only
(c) Upper triangular matrix only
(d) None of the above options are correct.

(v) The first-order forward difference approximation of $\left(\frac{\partial u}{\partial x}\right)_{i,j}$ using the Taylor series expansion
(a) $\frac{u_{i,j+1}-u_{i,j}}{2\Delta y}$ (b) $\frac{u_{i+1,j}-u_{i,j}}{2\Delta x}$ (c) $\frac{u_{i,j+1}-u_{i,j}}{\Delta y}$ (d) $\frac{u_{i+1,j}-u_{i,j}}{\Delta x}$

(vi) The first-order backward difference approximation of $\left(\frac{\partial u}{\partial x}\right)_{i,j}$ using the Taylor series expansion
(a) $\frac{u_{i,j}-u_{i,j-1}}{2\Delta y}$ (b) $\frac{u_{i,j}-u_{i,j-1}}{2\Delta x}$ (c) $\frac{u_{i,j}-u_{i,j-1}}{\Delta y}$ (d) $\frac{u_{i,j}-u_{i,j-1}}{\Delta x}$

(vii) The order and degree of the ODE $\frac{d^4y}{dx^4}+6\left(\frac{dy}{dx}\right)^4+3y=0$ are respectively
(a) 1; 4 (b) 4; 1 (c) 4; 4 (d) 1; 1

(viii) The simplest example of hyperbolic equation is

(a) $\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ (b) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

(c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (d) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$

(ix) In solving simultaneous equations by the Gauss-Jordan method, the coefficient matrix is reduced to

(a) Augmented matrix (b) Diagonal matrix

(c) Upper triangular matrix (d) Singular matrix

(x) The first-order central difference approximation of $\left(\frac{\partial u}{\partial x}\right)_{i,j}$ using the Taylor series expansion

(a) $\frac{u_{i,j+1}-u_{i,j-1}}{\Delta y}$ (b) $\frac{u_{i,j+1}-u_{i,j-1}}{2\Delta y}$ (c) $\frac{u_{i+1,j}-u_{i-1,j}}{\Delta x}$ (d) $\frac{u_{i+1,j}-u_{i-1,j}}{2\Delta x}$

Fill in the blanks with the correct word

(xi) The absolute error if the number $X = 0.00545828$ is rounded off to three decimal digits is ______________.

(xii) The decimal equivalent of the binary number $(110110)_2$ is _________.

(xiii) A square matrix in which $a_{ij} = 0$ for $i \neq j$ is called __________________.

(xiv) In Euler's method, if h is small the method is too slow; if h is large, the method gives inaccurate value: ______________ (True / False).

(xv) Using the trapezoidal rule with step size 1, the value of the definite integral $\int_1^6 \frac{dx}{1+x^2}$ is _____________.

Group - B

2. (a) Round off the numbers 865250 and 37.46235 to four significant figures and compute the absolute error E_a, the relative error E_r, and the percentage error E_p in each case. *[(CO1)(Apply/IOCQ)]*

(b) Find the root of the equation $\cos x = xe^x$ using the bisection method correct to four decimal places. *[(CO2)(Apply/IOCQ)]*

(2 + 2 + 1) + 7 = 12

3. (a) Find the root of the equation $\cos x = xe^x$ using the secant method. Continue the calculations till the fourth approximation. *[(CO2) (Apply/IOCQ)]*

(b) Find the positive root of $x^4 - x = 10$ using the Newton-Raphson method. Continue the calculations till the third approximation of the root. *[(CO2)(Apply/IOCQ)]*

6 + 6 = 12

Group - C

4. (a) Apply Cramer's rule to solve the questions

$3x + y + 2z = 3;\; 2x - 3y - z = -3;\; x + 2y + z = 4.$ *[(CO2) (Apply/IOCQ)]*

(b) Using the Newton's forward interpolation method, find the value of f(1.6) if:

x:	1	1.4	1.8	2.2
f(x):	3.49	4.82	5.96	6.5

[(CO3) (Apply/IOCQ)]

6 + 6 = 12

5. (a) A curve passes through the points (0, 18), (1, 10), (3, -18), and (6, 90). Find the slope of the curve at x = 2 using the Lagrange's interpolation formula. *[(CO3)(Apply/IOCQ)]*

(b) Apply the Gauss-Jordan method to solve the equations:
$x + y + z = 9;\ 2x - 3y + 4z = 13;\ 3x + 4y + 5z = 40.$ *[(CO2)(Apply/IOCQ)]*

6 + 6 = 12

Group - D

6. (a) Using Euler's method, find an approximate value of y corresponding to x = 0.15, given that $dy/dx = (y - x/y + x)$ and $y(x = 0) = 1.$ *[(CO5)(Apply/IOCQ)]*

(b) Use the trapezoidal rule for numerical integration to evaluate the definite integral $\int_0^1 x^3 dx$ considering five equal sub-intervals. *[(CO4) (Apply/IOCQ)]*

6 + 6 = 12

7. (a) Using the Picard's method of successive approximation, obtain a solution upto the third approximation of the equation $dy/dx = xy$ such that $y(x = 0) = 2.$ *[(CO5)(Apply/IOCQ)]*

(b) Using the Taylor series method, obtain the values of y at x = 0.1 and at x=0.2 by solving the equation $dy/dx = x + y$ such that $y\ (x = 0) = 2.$ *[(CO5)(Apply/IOCQ)]*

6 + 6 = 12

Group - E

8. (a) Classify the following equations as elliptic, hyperbolic or parabolic:

(i) $(1 + x^2)\frac{\partial^2 u}{\partial x^2} + (5 + 2x^2)\frac{\partial^2 u}{\partial x\, \partial t} + (4 + x^2)\frac{\partial^2 u}{\partial t^2} = 0$

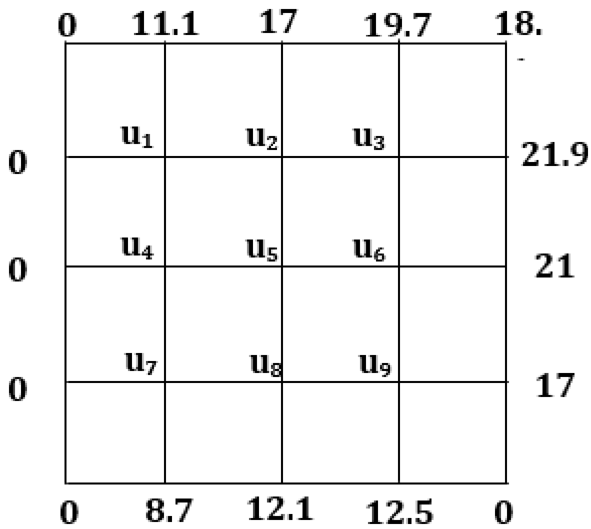
(ii) $\frac{\partial^2 f}{\partial x^2} + 2\frac{\partial^2 f}{\partial x\, \partial y} + \frac{\partial^2 f}{\partial y^2} = 0$

(iii) $y^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x\, \partial y} + x^2\frac{\partial^2 u}{\partial y^2} + 2\frac{\partial u}{\partial x} - 3u = 0$ *[(CO5)(Apply/IOCQ)]*

(b) In finite difference method, the central difference approximation $y'(x)$ is better thanthe forward or backward difference approximations: Justify. *[(CO6) (Understand/LOCQ)]*

(2 + 2 + 2) + 6 = 12

9. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for the square mesh with boundary values as shown in the following figure. Consider uniform step-size. Carry out computations till the fourth iteration level.



[(CO6) (Analyse/HOCQ)]

12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	6.25	81.25	12.5