# FIELD THEORY
## (ELEC 2204)

**Time Allotted : 2½ hrs** **Full Marks : 60**

***Figures out of the right margin indicate full marks.***

***Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.***

***Candidates are required to give answer in their own words as far as practicable.***

## Group – A

1. Answer any twelve: **12 × 1 = 12**

*Choose the correct alternative for the following*

(i) For ferromagnetic materials, the relative permeability is
(a) greater then 1 (b) lesser than 1 (c) equal to 1 (d) zero.

(ii) For diamagnetic materials, the relative permeability is
(a) greater then 1 (b) lesser than 1 (c) equal to 1 (d) zero.

(iii) Magnitude of Lorentz force experienced by a charge $q$ moving with a velocity $v$ in magnetic field $B$ is given by
(a) $q(v \times B)$ (b) $q(v.B)$ (c) $q \times (v \times B)$ (d) 0

(iv) A circular coil of radius 2m carries a current of 8 A. The value of magnetic field intensity at the centre is
(a) 1 A/m (b) 0.2 A/m (c) 0.5 A/m (d) 0 A/m

(v) Two long parallel conductors carry 100 A. If the conductors are separated by 20 mm, the force per metre of each conductor is
(a) 0.1 N/m (b) 0.5 N/m (c) 0.01 N/m (d) 0.2 N/m.

(vi) The result of the divergence of a vector field is
(a) a scalor (b) a vector
(c) may be a vector or a scalor depending on the field (d) always zero.

(vii) In a given region, for a scalar field V if $\nabla^2 V=0$, then the field is said to be
(a) solenoidal (b) harmonic (c) rotational (d) irrotational.

(viii) The relationship between $\overline{E}$ and V is
(a) $\overline{E} = -\nabla V$ (b) $\overline{E} = \nabla^2 V$ (c) $\overline{E} = -\nabla^2 V$ (d) $V = -\nabla \overline{E}$

(ix) $P = \int_V \overline{E}.\overline{J}\, dV$ is known as
(a) Gauss's law (b) Ohm's law (c) Joule's law (d) Continuity equation.

(x) Which one of the following statements is true?
(a) In lossless dielectric $\overline{E}$ and $\overline{H}$ are in time phase with each other
(b) In lossless dielectric $\overline{E}$ leands $\overline{H}$ by $90^{\circ}$
(c) In good conductors $\overline{E}$ leands $\overline{H}$ by $90^{\circ}$
(d) In good conductors $\overline{E}$ and $\overline{H}$ are in time phase with each other.

*Fill in the blanks with the correct word*

(xi) The coefficient of coupling is given by _____.

(xii) Magnitude of Lorentz force on a charge *q* moving with a velocity *v* in magnetic field *B* is given by _____.

(xiii) Poisson's equation is _____.

(xiv) The expression of $\overline{D}$ for a point charge Q is _____.

(xv) The expression of skin depth $\delta$ in terms of $\sigma$ is _____.

## Group - B

2. (a) Given the point A(r=6, $\theta=30^0$, $\varphi = 150^0$), determine the Cartesian coordinates of A. *[(CO1)(Evaluate/HOCQ)]*

(b) Write the expressions of differential areas in spherical coordinate system. *[(CO1)(Understand/LOCQ)]*

(c) Examine divergence theorem for the following field:
$\overline{A} = (x^2 - y)\hat{a}_x + (y^2 - z)\hat{a}_y + (z^2 - x)\hat{a}_z$ for a region defined by $0 \leq x \leq 1$, $0 \leq y \leq 2$ , $0 \leq z \leq 3$. *[(CO2)(Analyze/IOCQ)]*

**3 + 3 + 6 = 12**

3. (a) Write the expressions of differential length in Cartesian , cylindrical and spherical coordinates. *[(CO1)(Understand/LOCQ)]*

(b) Convert the given rectangular coordinates A(8,6,4) into corresponding cylindrical coordinates. *[(CO1)(Evaluate/HOCQ)]*

(c) Categorise vector fields by their divergence and curl. *[(CO2)(Analyze/IOCQ)]*

(d) Determine the Curl of the vector field at (2, 3, 4): $\vec{B} = \hat{a}_x + y^2\hat{a}_y$ *[(CO2)(Analyze/IOCQ)]*

**3 + 3 + 4 + 2 = 12**

## Group - C

4. (a) Show the derivation of the electric flux density $\vec{D}$ with the help of Gauss's Law at any point due to infinite sheet of charge. *[(CO3)(Understand/LOCQ)]*

(b) Assume $\vec{D} = (4xy^3z^2)\hat{a}_x + (2xy^2z^2)\hat{a}_y + (2xy^2)\hat{a}_z$ $pC/m^2$ in free space to calculate (i) total electrical flux passing through the surface x=3, $0 \leq y \leq 3$ , $0 \leq z \leq 3$ in a direction away from the origin. (ii) the total charge contained in a sphere of a radius 1mm centered at point P(2,3,4). *[(CO3)(Analyze/IOCQ)]*

(c) Conclude the relationship between $\vec{E}$ and V. The symbols have their usual meaning. *[(CO3)(Evaluate/IOCQ)]*

**4 + 5 + 3 = 12**

5. (a) How $P = I^2R$ can be derived from $V = \int_L \bar{E}.d\bar{l}$ and $I = \int_S \bar{J}.d\bar{s}$ ?
*[(CO3)(Remember/LOCQ)]*

(b) Develop boundary conditions of electric field for conductor-free space boundary. *[(CO3)(Apply/IOCQ)]*

(c) Let $\vec{E} = 12\hat{a}_x - 6\hat{a}_y + 7\hat{a}_z$ kV/m exists at conductor- free space boundary. Determine (i) $|\bar{E}|$ (ii)$E_N$ (iii) $E_T$ (iv) surface charge density $\rho_s$. *[(CO3)(Evaluate/HOCQ)]*

**4 + 5 + 3 = 12**

## Group - D

6. (a) Prove that the flux density at a distance *h* on a line passing through the centre of circular loop radius *a* and carrying current *I* is given by, $B = \frac{\mu_0 I a^2}{2(a^2+h^2)^{3/2}}$. The loop is in the XY plane and the line is perpendicular to the plane of the loop
*[(CO4)(Evaluate/HOCQ)]*

(b) State Ampere's Law. *[(CO4)(Remember/LOCQ)]*

(c) Using Ampere's Law, show that the magnetic field H at a radius r within a copper conductor carrying a current *I* is given by $H = \frac{I}{2\pi r}$, when $r > r_0$, where the radius of the cylindrical conductor is $r_0$ . Assume the current to be uniformly distributed throughout the conductor's cross-section. *[(CO4)(Evaluate/HOCQ)]*

(d) A steady current *I* flows in a conductor bent in the form of square. Determine the magnetic field intensity at the centre of the loop. The length of each side of the square loop is *a*. *[(CO4)(Evaluate/HOCQ)]*

(e) A toroid has a core of cross sectional area of 2500 $mm^2$, mean diameter 250 × $10^{-6}$ m and $\mu_r$ = 1000. Determine the number of turns to be wound on the core to obtain an inductance of 1 Henry. *[(CO4)(Evaluate/HOCQ)]*

**4 + 2 + 2 + 2 + 2= 12**

7. (a) Derive the expression of energy stored in Inductor or Magnetic Field.
*[(CO4)(Evaluate/HOCQ)]*

(b) What is inductance? *[(CO4)(Remember/LOCQ)]*

(c) Derive the expression of Inductance of a solenoid. *[(CO4)(Evaluate/HOCQ)]*

(d) A solenoid with 200 turns is 300 mm long and 20 mm in diameter. If the current is 600 mA, determine (i) Inductance (ii) energy stored in solenoid.
*[(CO4)(Evaluate/HOCQ)]*

(e) Write the first and second boundary conditions of magnetic field.
*[(CO4)(Remember/LOCQ)]*

**4 + 1 + 3 + 2 + 2 = 12**

## Group - E

8. (a) How is Maxwell's third equation in differential form derived from Faraday's Law? *[(CO5)(Remember/LOCQ)]*

(b) A straight conductor of 0.3 m lies on the x axis with one end at the origin. The conductor is subjected to a magnetic flux density $\vec{B} = 0.05\,\hat{a}_z$ Tesla and velocity $\vec{u} = 6\text{Sin}10t\,\hat{a}_y$ m/s. Calculate the motional electric field intensity and emf induced in the conductor. *[(CO5)(Apply/IOCQ)]*

(c) Identify the attenuation constant, phase constant, wavelength, intrinsic impedance and corresponding magnetic field while an electric field is represented by $\overline{E}_y = 20\,Cos(3\pi 10^8 t - \beta x)\hat{a}_y$ V/m is propagating at a frequency of 250 MHz through a medium having $\mu_r = 1$ and $\varepsilon_r = 75$. *[(CO6)(Apply/IOCQ)]*

**3 + 3 + 6 = 12**

9. (a) Assume a square loop of side a recedes with a uniform velocity $u_o\hat{a}_y$ from an infinite long filament carrying current I along $\hat{a}_z$ as shown in Fig. 1. Assuming $\rho = \rho_o$ at time $t = 0$, show that the emf induced in the loop at $t > 0$ is $V_{emf} = \frac{u_o a^2 \mu_o I}{2\pi\rho(\rho+a)}$

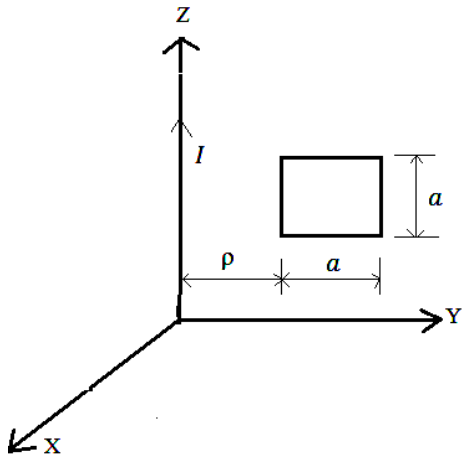


***Fig. 1***

*[(CO5)(Analyze/IOCQ)]*

(b) Develop the relationship between ac resistance and dc resistance. *[(CO6)(Apply/IOCQ)]*

(c) Develop the expressions of different parameters associated with a plane wave propagating in good conductors. *[(CO6)(Create/HOCQ)]*

**6 + 3 + 3 = 12**

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| Cognition Level | LOCQ | IOCQ | HOCQ |
|---|---|---|---|
| Percentage distribution | 22.90 | 44.80 | 32.30 |

**Course Outcome (CO):**

After the completion of the course students will be able to
CO1: Apply knowledge of different co-ordinate systems for field analysis problems.
CO2: Apply different techniques of vector calculus to analyze electromagnetic fields to reach substantiated conclusions.
CO 3: Solve static electric field problems for different engineering applications by using vector calculus.
CO4: Solve static magnetic field problems for different engineering applications by using vector calculus.
CO5: Apply the knowledge of Maxwell's equation in solving wave propagation problems.
CO6: Understand and analyze the concepts of electromagnetic waves.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*