PHYSICS - II
(PHYS 2101)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) When the external force on the system is zero then ______ remains constant?
(a) velocity of body (b) velocity of centre of mass
(c) both (a) and (b) (d) none of (a) and (b)

(ii) If the total external torque on a system of particles is zero then
(a) total linear momentum of the system is conserved
(b) total angular momentum of the system is conserved
(c) both (a) and (b)
(d) either (a) or (b).

(iii) The number of Lagrange equations for a system with 3 particles and 3 constraints is
(a) 1 (b) 5 (c) 6 (d) 9.

(iv) An Eulerian velocity field is given by $\vec{V} = x\hat{\imath} - 2y\hat{\jmath}$. The flow is
(a) steady and compressible (b) steady and incompressible
(c) unsteady and compressible (d) unsteady and incompressible.

(v) A simple pendulum in an accelerating lift is an example of a
(a) Scleronomic system (b) holonomic system
(c) gyroscopic system (d) nonholonomic system.

(vi) Which of the following constraints is holonomic?
(a) $x\dot{x} + \dot{y}^2 = 0$ (b) $\dot{x}sinkx + \dot{y}cosky = 0$
(c) $\dot{y} + x\dot{y} = 0$ (d) $\dot{x}sinkx + \dot{y}coskx = 0$.

(vii) If the normal mode vectors for a system are given by $\vec{v} = \begin{pmatrix}1\\1\end{pmatrix}$ and $\vec{w} = \begin{pmatrix}1\\-1\end{pmatrix}$, the possible kinetic energy matrix can be
(a) $\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$ (b) $\begin{pmatrix}2 & -1\\-1 & 2\end{pmatrix}$ (c) $\begin{pmatrix}1 & 2\\2 & 1\end{pmatrix}$ (d) $\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}$

(viii) For a three particle system with three holonomic constraints the number of Lagrange equation is

(a) 3 (b) 4 (c) 5 (d) 6.

(ix) The bulk modulus is related to the Young's modulus Y and Poisson's ratio ν through the following:

(a) $\frac{Y}{2(1+3\nu)}$ (b) $\frac{Y}{3(1+2\nu)}$ (c) $\frac{Y}{2(1-3\nu)}$ (d) $\frac{Y}{3(1-2\nu)}$

(x) The mutual ratios of all normal frequencies for a periodic system performing small oscillations are

(a) rational (b) irrational

(c) trascendental (d) all of these.

Fill in the blanks with the correct word

(xi) The continuity equation is a statement reflecting the conservation of ___________ in a fluid in motion.

(xii) The maximum number of normal frequencies for a three-particle spring-mass system is________.

(xiii) Lagrange equations determine the trajectories in __________ space.

(xiv) The number of constrained relations for a particle on an inclined plane is _____.

(xv) The momentum corresponding to a cyclic coordinate is __________.

Group - B

2. (a) Find the degrees of freedom of the following cases

i) Five particles moving in a plane

ii) Five particles moving in a space

iii) Two particles are connected by a rigid rod moving freely in a plane

[(CO1)(Understand/LOCQ)]

(b) Three equal point masses m located at (a,0,0) , (0,a,2a) and (0,2a,a). Find the principal moments of inertia about the origin and a set of principal axes.

[(CO1) (Apply/IOCQ)]

3 + (5 + 4) = 12

3. (a) Three particles of masses 2, 1, 3 respectively have position vectors $\vec{r}_1 = 5t\hat{i} - 2t^2\hat{j} + (3t-2)\hat{k}$, $\vec{r}_2 = (2t-3)\hat{i} + (12-5t^2)\hat{j} + (4+6t-3t^3)\hat{k}$ and $\vec{r}_3 = (2t-1)\hat{i} + (t^2+2)\hat{j} - t^3\hat{k}$ where t represents time.

Find (i) The velocity of the centre of mass at time t =1

(ii) The total linear momentum of the system at time t =1

[(CO1)(Understand/LOCQ)]

(b) Determine the directions of any one principal axes of the cube rotating about its corner.

[(CO1)(Evaluate/HOCQ)]

(3 + 3) + 6 = 12

Group - C

4. A particle of mass m is coming down along the length of a straight rigid wire that makes an angle α with respect to the vertical and rotating with a constant angular speed ω with respect to the vertical axis.
   (i) Write down the constraint relations for the system. *[(CO2)(Understand/LOCQ)]*
   (ii) Choose a suitable generalized coordinate q_1 for the system and construct the Lagrangian. *[(CO2)(Evaluate/HOCQ)]*
   (iii) Construct the Lagrange equation of motion. *[(CO2)(Create/HOCQ)]*
   (iv) Construct the Hamiltonian of the system. *[(CO2)(Create/HOCQ)]*

(3 + 3 + 3 + 3) = 12

5. (a) Show that if the Hamiltonian of a system is not an explicit function of time it is a conserved quantity. *[(CO2)(Understand/LOCQ)]*
   (b) A chain of length l and uniform mass density λ is hanging between two points on the same horizontal line in presence of constant gravity
      (i) Construct the action of the problem. *[(CO3)(Create/HOCQ)]*
      (ii) Find the equation of the curve representing the shape of the chain. *[(CO3)(Remember/LOCQ)]*
   (c) Show that if the Lagrangian of a system of n degrees of freedom is not an explicit function of time the Jacobi integral of the system is a conserved quantity. *[(CO2)(Understand/LOCQ)]*

3 + (3 + 3) + 3 = 12

Group - D

6. (a) Define SHM in the framework of Lagrangian mechanics. *[(CO4)(Remember/LOCQ)]*
   (b) The Lagrangian of a system is given by
   $L = \frac{1}{2}m\left(\dot{q}_1{}^2 + \dot{q}_2{}^2\right) - \frac{1}{2}k(q_1{}^2 + q_2{}^2 - q_1 q_2)$.
      (i) Construct the kinetic and potential energy matrices. *[(CO4)(Create/HOCQ)]*
      (ii) Construct the normal coordinates for the system. *[(CO4)(Create/HOCQ)]*
      (iii) Establish that normal coordinates are T-orthogonal. *[(CO4)(Apply/IOCQ)]*

2 + (4 + 3 + 3) = 12

7. A two-particle spring-mass system consisting of two equal masses of magnitude m are connected with a rigid supports with springs of spring constant k. They are connected by a coupling spring of spring constant k_0.
   (i) Construct the Lagrangian for the problem. *[(CO4)(Create/HOCQ)]*
   (ii) Explain the conditions for which small oscillation is possible for the system. *[(CO4)(Understand/LOCQ)]*
   (iii) Evaluate the normal frequencies of the system. *[(CO4)(Evaluate/HOCQ)]*
   (iv) Show that the normal mode vectors of such a system is T-orthogonal. *[(CO4)(Remember/LOCQ)]*

(3 + 3 + 3 + 3) = 12

Group - E

8. (a) State and elaborate the principle of superposition in elasticity. The Young's modulus and Poisson's ratio for steel are given by $210 \times 10^9\, N/m^2$ and 0.29 respectively. Calculate the bulk modulus and shear modulus of steel.

[(CO6)(Create/HOCQ)]

(b) A cylindrical aluminum rod of length 150 mm and cross section diameter 20 mm is subjected to a tensile force of 30 kN. The Young's modulus and Poisson ratio are given respectively by 70 GPa and 0.35. Determine (a) the increase in length of the rod and (b) the change in diameter of the rod. *[(CO6)(Evaluate/HOCQ)]*

(c) The following displacement field represents the deformation of a body in a given domain:

$$\vec{u} = \left[y^2\hat{\imath} + (3x+2)\hat{\jmath} + 2\hat{k}\right] \times 10^{-2} m$$

Determine the displacement of the point originally at the point $(1,1,4)m$ in the undeformed geometry?

[(CO4)(Evaluate/HOCQ)]

(2 + 2 + 2) + (2 + 2) + 2 = 12

9. (a) Develop the equation of continuity in differential form for an Eulerian system.

[(CO5)(Evaluate/HOCQ)]

(b) The velocity field for a flow in the xy-plane of a gas is given by

$u(x,y) = 4y/(x^2+y^2)$ and $v(x,y) = -4x/(x^2+y^2)$.

Show that this is an incompressible flow.

[(CO5)(Understand/LOCQ)]

(c) For a certain incompressible flow field it is suggested that the velocity components are given by the equations $u = 2xy$, $v = -x^2y$, $w = 0$. Is this a physically possible flow field? Explain. *[(CO5)(Understand/LOCQ)]*

(d) A velocity field is defined by $\vec{V} = 2x\hat{\imath} + 5yz^2\hat{\jmath} - t^2\hat{k}$. Find the divergence of the velocity field. Why is this an important quantity in fluid mechanics?

[(CO5)(Remember/LOCQ)]

4 + 3 + 3 + (1 + 1) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	38.54	12.50	48.96

Course Outcome (CO):

After the completion of the course students will be able to

1. Understand angular momentum kinetic energy and motion of a rigid body with applications in mechanical systems.
2. Understand calculus of variation as a core principle underlying majority of the physical laws: Newton's laws, Laplace equation (electrostatics and fluid mechanics), wave equation,heat conduction equation, control theory and many other.
3. Appreciate dynamical equations as a consequence of variational extremization of action functional along with the use of Euler-Lagrange equation to understand the behaviour of simple mechanical systems.
4. Appreciate the ubiquity of oscillation physics-from pendulum and spring-mass system to electrical circuit and movement of piston and comprehend the small motion of a system around stable equilibrium through the notion of normal modes—the meaning of eigen value problem in oscillation physics.
5. Elucidate the basic principles of fluid mechanics through the study of mass conservation, momentum balance, and energy conservation applied to fluids in motion.
6. Understand the mechanics of deformable bodies through a study of the concepts of normal and shear stresses and strains, following a review of the principles of statics.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*