DESIGN AND ANALYSIS OF ALGORITHMS
(CSBS 2203)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Time complexity of Floyd-Warshal algorithm to find the all pair shortest path of a given graph G(V,E)
(a) $O(V^3)$ (b) O(ElgV) (c) O(VlgV) (d) O(EV).

(ii) Which of the following set is NP-complete?
(a) Travelling salesman problem, Longest simple path problem
(b) Matrix-chain multiplication problem, Fraction knapsack problem
(c) Edge-cover problem, 2-CNF
(d) Finding all pair shortest path of a graph with negative edges.

(iii) Which of the following is the worst-case time complexity of BFS algorithm when the graph G = (V, E) is implemented by adjacency list?
(a) O(|V|+|E|) (b) $O(|V|^2)$ (c) O(|V|*|E|) (d) None of these.

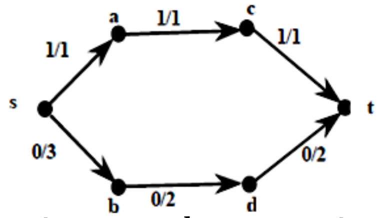
(iv) Dijkstra's Algorithm cannot be applied on
(a) directed and weighted graphs (b) graphs having negative weight function
(c) unweighted graphs (d) undirected and unweighted graphs.

(v) The number of edges in a DFS forest of a graph with 100 vertices and having 9 connected components is
(a) 99 (b) 90 (c) 91 (d) Less than 90.

(vi) The relationship between the number of back edges and the number of cycles in DFS is
(a) both are equal (b) back edges are half of cycles
(c) back edges are one quarter of cycles (d) none of these.

(vii) Consider 4 matrices M1, M2, M3 and M4 with dimensions 10 × 20, 20 × 50, 50 × 1 and 1 × 100 respectively. Which of the following fully parenthesized expression takes minimum number of scalar multiplications to multiply these 4 matrices?
(a) (M1 (M2 (M3M4))) (b) ((M1M2) (M3 M4))
(c) (M1 (M2M3) M4) (d) None of these.

(viii) The residual capacity of the augmenting path for the flow network shown below is
(a) 1 (b) 2 (c) 3 (d) 7.



(ix) For fractional knapsack problem, which of the following greedy strategies will always give you optimal solution?
(a) Select the object with largest profit first
(b) Select the objects in the order of non-decreasing weight
(c) Select the objects in the non-increasing order of profit per unit weight
(d) All of the above.

(x) Consider the following recurrence equation:

$$T(n) = \begin{cases} k & \text{for } n = 1, k \text{ is a constant} \\ 3T(n/2)+kn & \text{for } n>1, k \text{ is a constant} \end{cases}$$

Which of the following statements is TRUE?
(a) $T(n) = O(n^{1.59})$ (b) $O(1)$ (c) $O(n\log n)$ (d) None of these.

Group - B

2. (a) Define O – notation with suitable example. *[(CSBS2203.1)(Remember/LOCQ)]*

(b) Find the worst case time complexity of the following code snippet in asymptotic notation:

```
if (an array A of size n is sorted)
{
        for (i=0; i<n; i++) {…}
}
else {
        for (i=0; i<n/2; i++) {…}
}
```

[(CSBS2203.1)(Understand/LOCQ)]

(c) Write a recursive algorithm to compute X^n and show that the worst case time complexity of your algorithm is O (log n). *[(CSBS2203.4)(Apply/IOCQ)]*

2 + 2 + (5 + 3) = 12

3. (a) Is the array with values A = <5, 3, 17, 10, 84, 19, 6, 22, 9> a max-heap? If not, then illustrate the operation of BUILD-MAX-HEAP on the given array A to create the equivalent max-heap. *[(CSBS2203.3)(Apply/IOCQ)]*

(b) What is the running time requirement of sorting the elements of an array A of size n in increasing order using HEAPSORT, when the array A is already sorted in increasing order? Give proper reasons. (The user is not aware that array A is already sorted). *[(CSBS2203.3)(Analyse/IOCQ)]*

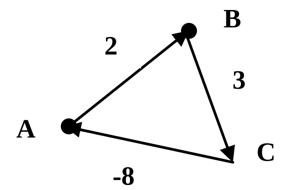
(c) Heapsort and Mergesort have the same worst-case time complexities. Is there any reason why Heapsort may be considered better compared to Mergesort? *[(CSBS2203.2)(Evaluate/HOCQ)]*

(1 + 6) + 3 + 2 = 12

Group - C

4. (a) Prove that sub-paths of shortest paths are also shortest paths. *[(CSBS2203.3)(Apply/IOCQ)]*

(b) What problem do you face when you apply Bellman Ford and Dijkstra's algorithms on the following graph? Give proper reasons for each case. *[(CSBS2203.2)(Analyse/IOCQ)]*



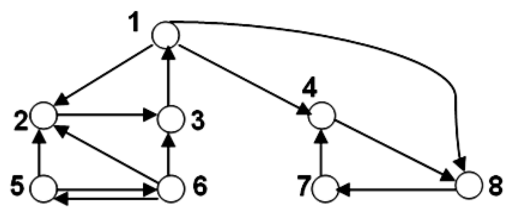
(c) You have the following input for the fractional-knapsack problem:
Capacity of the knapsack = 50

Item	Profit	Weight
1	100	10
2	60	15
3	360	30

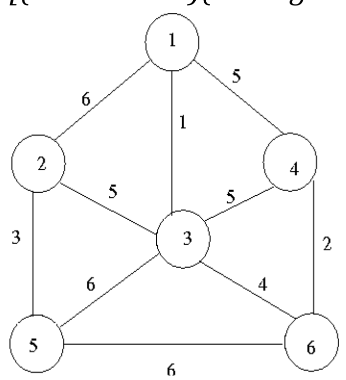
What will be the profit? Show the proportions by which each item will be chosen. *[(CSBS2203.2)(Distinguish/LOCQ)]*

3 + 3 + 6 = 12

5. (a) Consider the following graph:
Illustrate the progress of the DFS algorithm on the above graph, when the starting node is 1. Also identify the tree edges, back edges and forward edges. *[(CSBS2203.3)(Apply/IOCQ)]*



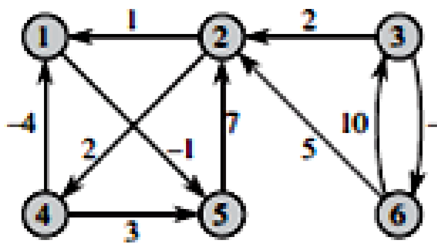
(b) Find out the Minimum Spanning Tree of the following graph using Kruskal's algorithm with source vertex '1'. *[(CSBS2203.2)(Distinguish/LOCQ)],[(CSBS2203.4)(Apply/IOCQ)]*



(5 + 2) + 5 = 12

Group - D

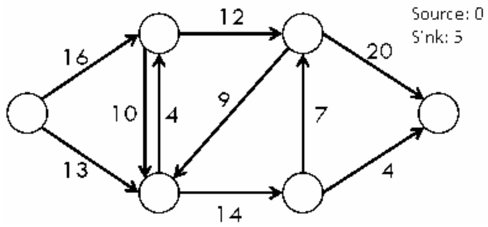
6. (a) Consider the following directed graph:
Apply Floyd-Warshall algorithm on the above graph to find the shortest path between every pair of vertices. In each iteration, you are supposed to show D $^{(k)}$ and Π $^{(k)}$ matrices, for 0<=k<=6, where D matrix stores the weight of the shortest path between every pair of vertices, and Π is the predecessor matrix. *[(CSBS2203.4)(Apply/IOCQ)]*



(b) What is the time complexity of Floyd Warshal algorithm to find all pair shortest paths of a given graph G(V,E)? Justify your answer. *[(CSBS2203.2)(Distinguish/LOCQ)]*

8 + (2 + 2) = 12

7. (a) Using Ford Fulkerson algorithm, find out the amount of maximum flow of the below flow network. *[(CSBS2203.2)(Distinguish/LOCQ)]* *[(CSBS2203.4)(Apply/IOCQ)]*



(b) What is an s-t cut of a flow network? What is Max-flow Min-cut theorem? What will be the Min-cut of the above graph after running Ford Fulkerson algorithm? *[(CSBS2203.4)(Apply/IOCQ)]*

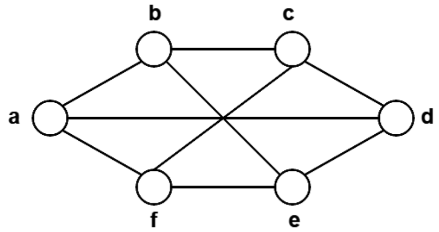
6 + (1.5 + 2 + 2.5) = 12

Group - E

8. (a) Define the complexity class NP. Differentiate between NP-Complete and NP-Hard problems. *[(CSBS2203.5)(Remember/LOCQ)]*

(b) Show that CLIQUE problem is NP-Complete. *[(CSBS2203.5)(Apply/IOCQ)]*

(c) What will be the chromatic number of the following graph?



(2 + 2) + 5 + 3 = 12

9. (a) Define the following terms in the context of optimization problems —
(i) Approximation scheme (ii) Polynomial-time approximation scheme
(iii) Fully polynomial-time approximation scheme (FPTAS). *[(CSBS2203.5)(Remember/LOCQ)]*

(b) Write a polynomial-time 2-approximation algorithm for the Travelling Salesman problem. *[(CSBS2203.5)(Create/HOCQ)]*

(c) Show how backtracking algorithm can be used to solve 4-queens problem. *[(CSBS2203.4)(Apply/IOCQ)]*

3 + 4 + 5 = 12

Cognition Level	*LOCQ*	*IOCQ*	*HOCQ*
Percentage distribution	*22*	*72*	*6*

Course Outcome (CO):

After the completion of the course students will be able to

CSBS2203.1. Analyze the time and space complexity of an algorithm using asymptotic notations.
CSBS2203.2. Distinguish the algorithms to solve optimization problems.
CSBS2203.3. Illustrate the suitable algorithmic technique to find out a solution of decision making problems.
CSBS2203.4. Apply the appropriate algorithmic techniques to solve the mathematical puzzles.
CSBS2203.5. Describe different NP-hard and NP-complete problems and necessity of approximation algorithms.
CSBS2203.6. Develop algorithms to solve common engineering problems.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*