PHYSICS-II
(PHYS 2211)

Time Allotted: 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) For a system in thermal contact with a heat reservoir and in which the volume is held constant, spontaneous changes in the system will occur in the direction of decreasing
(a) Helmholtz free energy (b) Enthalpy
(c) Gibbs free energy (d) Internal energy.

(ii) For a system in thermal contact with a heat reservoir and in which the pressure is held constant, spontaneous changes in the system will occur in the direction of decreasing
(a) Helmholtz free energy (b) Enthalpy
(c) Gibbs free energy (d) Internal energy.

(iii) In an ideal gas, the internal energy is a function of
(a) Pressure only (b) Volume only
(c) Temperature only (d) Both pressure and volume.

(iv) The nth energy eigenvalue for a particle in a 1-D box $(0 < x < l)$ is proportional to
$(a)\ l$ (b) l^2
(c) l^{-2} (d) l^{-1}

(v) If the operator A corresponds to a conserved quantity for a system with Hamiltonian H,
(a) $AH + HA = 0$ (b) $AH - HA = 0$
(c) $[A,H] = 1$ (d) $[A,H] = -1$

(vi) Dimension of reciprocal lattice vector is
(a) $[L]^{-1}$ (b) $[L]^0$
(c) [L] (d) $[L]^2$

(vii) If $\vec{a}_i$ is primitive basis vector of a unit cell in direct lattice, and $\vec{b}_i$ is that in the corresponding reciprocal lattice then

(a) $\vec{a}_i.\vec{b}_j = o$ (b) $\frac{|\vec{a}_i|}{|\vec{b}_j|} = 2\pi\delta_{ij}$

(c) $\frac{|\vec{a}_i|}{|\vec{b}_j|} = e^{-i\,2\pi\delta_{ij}}$ (d) $\vec{a}_i.\vec{b}_j = 2\pi\delta_{ij}$

(viii) Number of 2-fold symmetry axes in a cubic cell is

(a) 0 (b) 3

(c) 4 (d) 6

(ix) In an (E-K) curve , the condition for energy discontinuity is

(a) $k = \pm\frac{n\pi}{a}$ (b) $k = \frac{2a}{n\pi}$

(c) $k = \pm\frac{a}{n\pi}$ (d) $k = \frac{n\pi}{a}$

(x) Bloch theorem is applicable to

(a) periodic potential (b) constant potential

(c) infinite potential (d) none of the above

Group – B

2. (a) For any system with state variables P, V, T, show that

$$\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial P}\right)_T \left(\frac{\partial T}{\partial V}\right)_P = -1$$

P, V, T are the pressure, volume and temperature respectively.

(b) Verify that the ideal gas equation of state satisfies the above equation.

(c) Show that during a reversible, adiabatic expansion of an ideal gas,
$TV^{\gamma-1} = a\ constant$
where $\gamma = \frac{C_P}{C_V}$.

(d) Plot the Carnot cycle for an ideal gas. Label the processes involved. Heat is supplied to an engine at the rate of $20{,}000\ Joules/s$ and the engine has an output of $10{,}000\ Watts$. What is the efficiency of the engine?

3 + 2 + 3 + (2 + 2) = 12

3. (a) State the Kelvin-Planck and Clausius formulations of the second law of thermodynamics. Show that the falsity of the Clausius statement implies the falsity of the Kelvin-Planck statement.

(b) Evaluate the entropy change of an ideal gas undergoing a free expansion doubling its volume from V to $2V$.

(c) Starting from their definitions, develop the expressions of the differentials of the Helmholtz and Gibbs free energies. Using these, develop the two corresponding Maxwell relations.

(d) The Gibbs free energy of one mole of a certain gas at temperature T and pressure P is given by

$$G = RTlnP + A + BP + {}^{CP^2}/_{2} + {}^{DP^3}/_{3}$$

where A, B, C, D are constants, R is the gas constant. Find the equation of state of the gas.

(2 + 2) + 2 + (2 + 2) + 2 = 12

Group – C

4. (a) The Hamiltonian of a system is given by $H = \begin{pmatrix} 2 & -i\sqrt{5} \\ i\sqrt{5} & -2 \end{pmatrix}$. Construct the time-dependent Schrodinger equation for the system if it acts on a state $\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$. Evaluate the energy eigenvalues.

   (b) Momentum operator is the generator of time translation. Explain.

   (c) The quantum state of a system is given by $\frac{1}{5}|u\rangle + \frac{2\sqrt{6}}{5}|v\rangle$. Identify the state where the probability of finding the system is higher?

(4 + 3) + 2 + 3 = 12

5. (a) Given $[A, B] = 1$, Show that $[A, B^2] = 2B$.

   (b) The Hamiltonian of a system is given by $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. Evaluate the energy eigenvalues and normalized eigenstates.

   (c) The possible quantum states of a system are $|p\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $|q\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Evaluate the expectation value of an operator $T = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ in the state $|p\rangle$.

3 + (3 + 3) + 3 = 12

Group – D

6. (a) Show (101), $(11\bar{2})$ planes in Cartesian system by drawing a unit cell in the first quadrant in each case.

   (b) Determine linear density of atoms of Ba along [101] and [111] direction in bcc Ba. Atomic volume of Ba is 39.24 cm^3/mole.

   (c) Atomic volume of Cu is 7.1 × 10^{-6} m^3/mole. Find out the interplanar spacing of (110) planes of fcc Cu.

   (d) Show all 4-fold symmetry axes by drawing a cubic unit cell.

(2 + 2) + (2 + 2) + 2 + 2 = 12

7. (a) Why the diffraction pattern does not contain lines such as (100), (300), (111) or (210) for the bcc lattice?

   (b) Show that the reciprocal lattice vector $\vec{G} = h\vec{b_1} + k\vec{b_2} + l\vec{b_3}$ is perpendicular to (*hkl*) plane, where $\vec{b_1}$,$\vec{b_2}$and $\vec{b_3}$arethree reciprocal primitive vectors. Show that the distance between two adjacent parallel planes of the lattice is $2\pi/|\vec{G}|$.

   (c) Develop the equation $2d \sin\theta = n\lambda$ from Bragg condition written in terms of wave vector (X-ray) and reciprocal lattice vector.

3 + (3 + 3) + 3 = 12

Group – E

8. (a) Write down the differential equation of motion of the nth particle of n dimensional lattice array in one dimension mentioning all the assumptions you make.
   (b) Solve the above said differential equation and establish the dispersion relation for it.
   (c) Find the group velocity at high frequency.

(2 + 2) + (2 + 2) + 4 = 12

9. (a) Plot the graph of $P\frac{sin\alpha a}{\alpha a}+\cos\alpha a=\cos\alpha a$ Vs αa where $\alpha^2=\left(\frac{2mE}{\hbar^2}\right).$, a be the periodicity of the lattice and symbols have their usual meanings.
   (b) Explain the graph and comment on energy of the electron for P$\rightarrow$0 and P$\rightarrow\infty$.

4 + (4 + 4) = 12