FINITE ELEMENT METHOD
(MECH 3142)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The eminent personality who left remarkable contribution in setting up Finite Element Method to solve any physical problem
(a) Boris Galerkin (b) Claude-Louis Navier
(c) Sir Gorge Stokes (d) Heisenberg.

(ii) The Field Variable are the
(a) Independent Variables
(b) Neither dependent variable nor independent variable.
(c) Dependent variable
(d) Domain variables.

(iii) For deriving solution function from Governing equation using 'Principle of Total Stationary Potential (PSTP)' following method is used
(a) Double Integration Method (b) Area Moment Method
(c) Rayleigh Ritz Method (d) Navier Stokes Method.

(iv) Linear strain ε_x for a beam is expressed as
(a) $\varepsilon_x = -y\frac{d^2v}{dx^2}$ (b) $\varepsilon_x = -y\frac{dv}{dx}$
(c) $\varepsilon_x = -y\frac{d^3v}{dx^3}$ (d) $\varepsilon_x = -\frac{d^2v}{dx^2}$.

(v) BAR element has two shape functions because a BAR element has
(a) Two nodal DOF
(b) Two elemental DOF
(c) Two nodes
(d) All element is having two shape functions.

(vi) Stiffness matrix of a BEAM element is a

(a) 3 × 3 matrix (b) 4 × 4 Matrix

(b) 2 × 2 matrix (d) Single column matrix.

(vii) The coefficient in stress-strain relation for a linear, elastic, isotropic material under plane strain condition is given by- (CO 4)

(a) $\frac{E}{(1+\nu)}\begin{bmatrix}1-\nu & 0 & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}$ (b) $\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}$

(c) $\frac{E}{(1-2\nu)}\begin{bmatrix}1-\nu & 0 & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}$ (d) $\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}\nu & 0 & 0\\ \nu & \nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}$

(viii) CST element possesses.

(a) Constant field variable throughout the element

(b) Derivative of the field variable is constant throughout the element

(c) Variation of the field variable is quadratic throughout the element

(d) Variation of the field variable is cubic throughout the element.

(ix) For a Four-Noded Rectangular element if one of the shape functions in user-coordinate (x, y) is expressed as $N_1 = \left(1-\frac{x}{l}\right)\left(1-\frac{y}{w}\right)$ then the corresponding shape function in normalized coordinate (ξ, η) will be?

(a) $\frac{1}{4}(1-\xi)(1+\eta)$ (b) $\frac{1}{4}(1-\xi)(1-\eta)$

(c) $\frac{1}{2}(1-\xi)(1-\eta)$ (d) $\frac{1}{4}(1-\xi)$.

(x) The sequence of the numerical simulation in any FEA software is

(a) Pre-processing→Solution→Post-processing

(b) Post-processing→Solution→Pre-processing

(b) Pre-processing→Post-processing→Solution

(d) Solution→ Pre-processing→Post-processing.

Group – B

2. Using Galerkin's Method of Weighted Residual, obtain a two term solution of the differential equation $\frac{d^2y}{dx^2}$ + 20x^2 = 10; $0 \le x \le 1$ and boundary conditions y(0)=0, y(1)=0. Use trial functions $N_1(x) = x(x-1)$ and $N_2(x) = x^2(x-1)$

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3. A simply supported beam is subjected to a uniformly distributed load as shown in fig: 1 below. Given

The strain energy U = $\int_0^L \frac{1}{2} EI\left(\frac{d^2v}{dx^2}\right)^2 dx$

And potential of the external force is V = $-\int_0^L q_0\, v\, dx$

Solve the displacement field assuming it as $v(x) = c_1 \times \sin\frac{\pi x}{L}$ by Rayleigh-Ritz method.

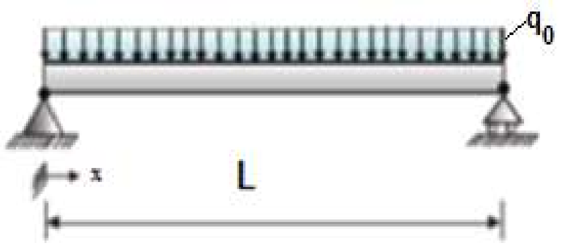

Fig: 1

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Group – C

4. Consider the spring assembly system shown in fig: 2 below. Using finite element method (FEM) find out the global stiffness matrix and then find out deflections at node 2, 3 and 4.

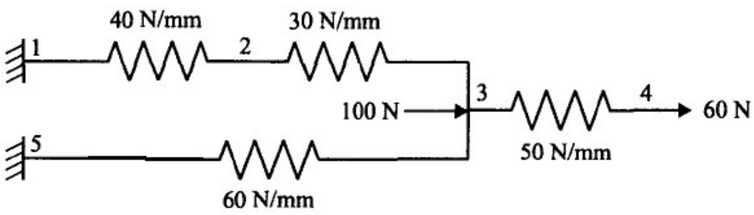

Fig: 2

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5. The plane truss shown in Figure 5 below is composed of members having a square 15 mm × 15 mm cross section and modulus of elasticity E = 69 GPa. Determine the global stiffness matrix and write down the FEA formulation showing the nodal forces.

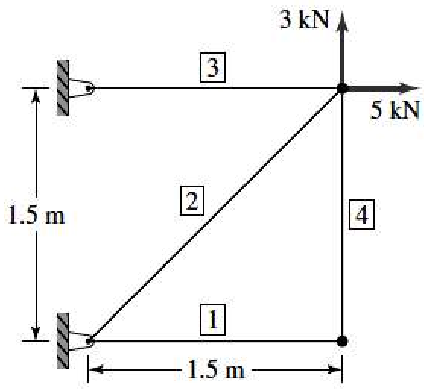

Fig 5

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Group – D

6. Evaluate the given integral analytically and using Gauss-Legendre formula.
$\int_{-2}^{8}\int_{2}^{10}(y^3 + 3y^2 + 5y + 2x^2 + x + 10)dxdy.$

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7. What do you understand by Isoparametric element? Present a brief but to-the-point discussion on your understanding.
Consider the isoperimetric quadrilateral element in Figure below. Map the point $\xi = 0.5$, $\eta = 0.7$ in the parent element to the corresponding physical point in the quadrilateral element.

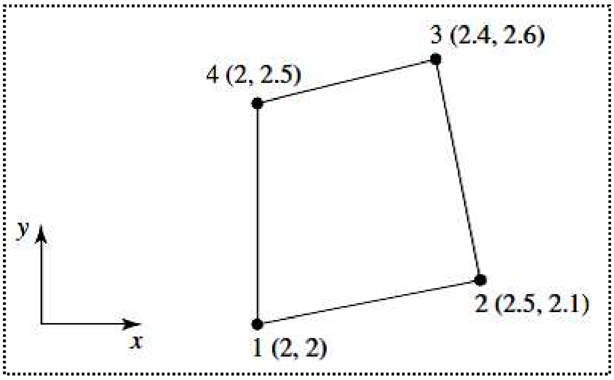

Fig: 7

4 + 8 = 12

Group – E

8. Describe in detail about full classification of elements on the basis of topology used in a FEA software.

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9. Discuss in detail about the methodology to evaluate stress in a beam using Finite Element Method.

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Department & Section	Submission link:
ME	https://classroom.google.com/c/Mjc0NzQxNTQyNzQ4/a/Mjc0NzQxNTQyODY3/details