PHYSICS - II
(PHYS 2101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The numbers of degrees of freedom of 3 particles moving in plane so that the distance between any two of them is always constant is
(a) 3 (b) 6 (c) 2 (d) 4.

(ii) For a holonomic system with n degrees of freedom the number Lagrange equations would be
(a) $2n$ (b) $2n+1$ (c) $3n$ (d) $2n-1$

(iii) The Euler equation is
(a) the continuity equation for a fluid
(b) the momentum equation for an inviscid fluid
(c) the momentum equation for a viscous fluid
(d) none of the above.

(iv) Five equal point masses are placed at the five corners of a square pyramid whose square base is centered on the origin in the X-Y plane, with side L and whose apex is on the Z axis at a height H above the origin. The centre of mass of the five mass system will be at
(a) (0,0,H/5) (b) (0,0,H/4) (c) (0,0,H/3) (d) (0,0,H/2).

(v) Hamilton's equations of motion determines the trajectory in
(a) configuration space (b) Hilbert space
(c) phase space (d) Fock space.

(vi) The stress tensor in two dimensions has
(a) Nine components of which three are independent
(b) Nine components of which six are independent
(c) Six components of which three are independent
(d) Four components of which two are independent.

(vii) A pendulum is oscillating in a vertical plane. The dimension of its phase space is
(a) 1 (b) 2 (c) 3 (d) 4.

(viii) Three equal point masses m are located at (a,0,0), (0,a,2a) and (0,2a,a). Which one is the value of I_{xx}?
(a) $10ma^2$ (b) $6ma^2$ (c) ma (d) $2ma^2$

(ix) For a system performing small oscillation around any point of equilibrium the potential energy matrix is
(a) positive definite (b) negative definite
(c) semi-definite (d) indefinite.

(x) An Eulerian velocity field is given by $\vec{V} = x\hat{\imath} + 2t\hat{\jmath}$. The flow is
(a) Steady and compressible (b) Steady and incompressible
(c) Unsteady and compressible (d) Unsteady and incompressible.

Group – B

2. (a) Show that, for a system with two particles of mass m_1 and m_2 such that $m_2 > m_1$, prove that the distances of CM from $m_1$and m_2 are in the ratio of $\frac{m_2}{m_1}$.

(b) Find the centre of mass of a solid square pyramid having its square base centered in the xy plane.

(c) Prove that the total angular momentum of a system of particles about any point 'O' equals the angular momentum of the total mass assumed to be located at the center of mass plus the angular momentum of motion about the center of mass.

(d) Calculate the moment of inertia and products of inertia for rotation about Z axis with angular velocity $\vec{\omega}$, of two particles of equal mass m located at $(0, y_0, z_0)$ and $(0, -y_0, z_0)$. Is angular momentum in the same direction as angular velocity?

3 + 3 + 3+ 3 = 12

3. (a) Establish the relation between moments of inertia about two parallel set of axes (X,Y,Z) and (X′,Y′,Z′) respectively, such that one set of axes passes through the centre of mass (at point O) of the rigid body and the origin of the other set is shifted to point O′, where OO′=$\vec{a}$.

(b) Three equal point masses m are located at (a,0,0), (0,a,2a) and (0,2a,a). Find the inertia tensor and principal moments of inertia about the origin and set a principal axes.

(c) Write down the Euler's equation for a rotating rigid body.

3 + (2 + 2 + 3) + 2 = 12

Group – C

4. (a) A pendulum bob of mass m is hanging with an inextensible string from the ceiling of a car moving with constant velocity v along the horizontal direction. If the string length is l

(i) Write down the constraint relations for the system.
(ii) Construct the Lagrangian for the system
(iii) Find the time period of oscillation.

(b) The Lagrangian of a spring mass system is given by
$L = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2$
Construct the Hamiltonian of the system and Hamilton equations of motion. What do you mean by phase space of a system with n degrees of freedom?

(2 + 2 + 2) + (2 + 2 + 2) = 12

5. (a) Write down relation between action A and Lagrangian L of a typical variational problem. Show that the brachistochrone under constant gravity is a cycloid.

(b) Show that if the Hamiltonian is not an explicit function of time it is a conserved quantity.

(c) The lagrangian of a system is given by
$L = \frac{1}{2}m\left(\dot{q}_1{}^2 + \dot{q}_2{}^2 + \dot{q}_3{}^2\right) - k(q_1 q_2)$. Find out the components of generalized momenta.

(2 + 4) + 3 + 3 = 12

Group – D

6. The kinetic and potential energies of a system are given by $T = \frac{1}{2}\dot{q}_1{}^2 + 2\dot{q}_2{}^2$ and $V = \frac{5}{2}q_1{}^2 + 10q_2{}^2 - kq_1q_2$
(i) Find out the condition obeyed by k so that the system can show small oscillation.
(ii) Find the normal frequencies and normal coordinates of the system for $k = 3$.

(4 + 4 + 4) = 12

7. (a) Define Simple Harmonic motion in the sense of Lagrangian dynamics. Find out the points of equilibrium of the potential $V(x) = \frac{x^3}{3} - \frac{3}{2}x^2 + 2x$. Write down the condition obeyed by the potential $V(x)$to produce Simple Harmonic Motion.

(b) A system has an equation of motion $\ddot{x} + \beta^2 x = 0$. Show that it is a conservative system.

(c) A system has two normal frequencies ω_1 and ω_2. Write down the condition for which the system would be periodic.

(2 + 2 + 2) + 4 + 2 = 12

Group – E

8. (a) State and explain the principle of superposition in elasticity.

(b) The Young's modulus and Poisson's ratio for steel are given by $210 \times 10^9\ N/m^2$ and 0.29 respectively. Calculate the bulk modulus and shear modulus of steel.

(c) Consider a cube of side l. Now suppose the cube is subjected to vertical compressive forces F on its top and bottom faces and equal horizontal stretching forces on two opposite side faces. No external forces are acting on the remaining two faces of the cube. Calculate the total strains in the vertical and horizontal directions.

(d) Consider a rectangular block which is stretched along its length, but lateral contractions are not allowed. Calculate the effective Young's modulus of the block.

(e) A sample of a homogeneous and isotropic material is deformed under plane strain conditions. Derive the expressions for the normal strains only.

2 + 3 + 3 + 2 + 2 = 12

9. (a) A velocity field is given by $\vec{V} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$ where the field, in general, is a function of space and time. Show that Du/Dt can be written as $\vec{V}.\vec{\nabla}u$ for a steady flow. Derive the corresponding expression for $D\vec{V}/Dt$.

(b) Given the Eulerian velocity field $\vec{V} = 2t\hat{\imath} + xz\hat{\jmath} - t^2y\hat{k}$, find the acceleration following the fluid particle. Determine if the fluid is incompressible.

(c) Write down the integral form of the equation of continuity. Apply this equation for a steady one- dimensional flow through a diverging duct.

(2 + 2) + (2 + 2) + (2 + 2) = 12

Department & Section	Submission Link
ME A	https://classroom.google.com/c/MTI5NTg2OTA5NDA0/a/Mjc0MDU3NTk5NjMz/details
ME B	https://classroom.google.com/c/MTQyMzI3Nzc0Mzk5/a/Mjc0MDU3NTk5NzUx/details