INTRODUCTION TO OPTIMIZATION
(MATH 2102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A feasible solution to an LP problem
(a) must satisfy all of the problem's constraints simultaneously
(b) need not satisfy all of the constraints, only some of them
(c) must be a corner point of the feasible region
(d) must optimize the value of the objective function.

(ii) If a negative value appears in the solution values column of the simplex table, then
(a) the solution is optimal
(b) the solution is infeasible
(c) the solution is unbounded
(d) the formulation of the LPP is incorrect.

(iii) To convert ≥ inequality constraints into equality constraints, we must
(a) add a surplus variable
(b) subtract an artificial variable
(c) subtract a surplus variable and an artificial variable
(d) add a surplus variable and subtract an artificial variable.

(iv) The initial solution of a transportation problem can be obtained by applying any known method. However, the only condition is that
(a) the solution be optimal (b) the rim conditions are satisfied
(c) the solution not be degenerate (d) none of the above.

(v) In a Branch and Bound minimization tree, the lower bounds on objective function value
(a) do not decrease (b) do not increase
(c) remain constant (d) oscillates.

(vi) If there are n jobs to be performed, one at a time, on each m machines, the possible sequences would be
(a) $(n!)^m$ (b) $(m!)^n$ (c) $(n)^m$ (d) $(m)^n$

(vii) A saddle point exists when
(a) maximin value = maximax value (b) minimax value = maximax value
(c) minimax value = maximin value (d) minimax value ≠ maximin value.

(viii) Two persons zero-sum game means that
(a) the sum of the losses to one player is equal to the sum of the gains to the other.
(b) the sum of the losses to one player is not equal to the sum of the gains to the other.
(c) both (a) and (b).
(d) the game does not have a saddle point.

(ix) For the game with the given pay off matrix, the value of the game is

PLAYER B

PLAYER A			
	−1	2	−2
	6	4	−6

(a) 2 (b)0 (c) −2 (d) 1.

(x) Total elapsed time to process all jobs through two machines is given by
(a) $\sum_{j=1}^{n} M_{1j} + \sum_{j=1}^{n} M_{2j}$ (b) $\sum_{j=1}^{n} M_{2j} + \sum_{j=1}^{n} I_{2j}$.
(c) $\sum_{j=1}^{n} (M_{2j} + I_{1j})$ (d) $\sum_{j=2}^{n} (M_{2j} + I_{12})$.

Group – B

2. (a) Prove that $B = \{(x_1, x_2): x_1^2 + x_2^2 \leq 9\}$ is a convex set. [(MATH2102.1, MATH2102.4)(Understand/LOCQ)]

(b) Use the Simplex method to find the maximum value of

$$Z = 3x_1 + x_2 + 3x_3,$$

subject to the constraints

$$2x_1 + x_2 + x_3 \leq 2,$$
$$x_1 + 2x_2 + 3x_3 \leq 5,$$
$$2x_1 + 2x_2 + x_3 \leq 6,$$

where $x_1, x_2, x_3 \geq 0.$

[(MATH2102.1, MATH2102.4)(Evaluate/HOCQ)]

4 + 8 = 12

3. (a) Write the dual of the following LPP

$$Min\ Z = 2x_2 + 5x_3$$

subject to the constraints

$$x_1 + x_2 \geq 2,$$
$$2x_1 + x_2 + 6x_3 \leq 6,$$
$$x_1 - x_2 + 3x_3 = 4,$$

where $x_1, x_2, x_3 \geq 0.$

[(MATH2102.1, MATH2102.4)(Understand/IOCQ)]

(b) Use Simplex method to solve the following LPP

$$Max\ Z = 2x_1 - 3x_2,$$

subject to

$$-x_1 + x_2 \geq -2,$$
$$5x_1 + 4x_2 \leq 46,$$
$$7x_1 + 2x_2 \geq 32,$$

where $x_1, x_2 \geq 0.$

[(MATH2102.1, MATH2102.4)(Evaluate/HOCQ)]

4 + 8 = 12

Group – C

4. (a) Consider the following transportation problem and develop a linear programming (LP) model.

	D1	D2	D3	D4	Supply (Availability)
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand (Requirement)	5	8	7	14	

[(MATH2102.1, MATH2102.2)(Understand/LOCQ)]

(b) Solve the following all integer programming problem using the branch and bound method.

$$\text{Maximize } Z = 2x_1 + 3x_2$$

subject to the constraints

$$6x_1 + 5x_2 \leq 25,$$
$$x_1 + 3x_2 \leq 10,$$

where $x_1, x_2 \geq 0.$ [(MATH2102.1MATH2102.2)(Evaluate/HOCQ)]

4 + 8 = 12

5. (a) Find a minimum cost of the following Transportation problem using north-west corner method and hence perform the optimality test.

	D1	D2	D3	D4	Supply (Availability)
S1	10	7	3	6	3
S2	1	6	8	3	5
S3	7	4	5	3	7
Demand (Requirement)	3	2	6	4	

[(MATH2102.1MATH2102.2)(Apply/IOCQ)]

(b) Find the optimal assignment cost from the following cost matrices:

	A	B	C	D
I	9	12	10	10
II	8	12	7	9
III	4	6	7	8
IV	8	4	5	5

[(MATH2102.1MATH210.2)(Apply/IOCQ)]

7 + 5 = 12

Group – D

6. (a) Use dominance principle to solve the following game:

		PLAYER B	
PLAYER A	−1	2	−1
	1	−2	5
	−5	2	−5

[(MATH2102.1, MATH2102.3)(Evaluate/HOCQ)]

(b) Solve the following game graphically:

	PLAYER B	
PLAYER A	2	−3
	−2	5
	0	−1

[(MATH2102.1, MATH2102.3)(Apply/IOCQ)]

6 + 6 = 12

7. (a) Find the ranges of value of P and Q which will render the entry (2,2) a saddle point for the following game.

	Player B		
Player A	2	4	5
	10	7	Q
	4	P	6

[(MATH2111.1, MATH2111.3)(Analyze/IOCQ)]

(b) Use algebraic method to solve the following game.

	Player B		
Player A x_1	−1	2	1
x_2	1	−2	2
x_3	3	4	−3
	y_1	y_2	y_3

[(MATH2111.1, MATH2111.3)(Apply/IOCQ)]

6 + 6 = 12

Group – E

8. (a) Five jobs are performed, first on machine X and then on machine Y. The time taken, in hours by each job on each machine is given below:

Job :	A	B	C	D	E
Time on machine X :	12	4	20	14	22
Time on machine Y :	6	14	16	18	10

Determine the optimum sequence of jobs that minimizes the total elapsed time to complete the jobs. Also compute the minimum time. [(MATH2111.1, MATH2111.5, MATH2111.6)(Apply/IOCQ)]

(b) We have five jobs, each of which must go through the machines A, B and C in the order ABC. Processing times (in hours) is as follows:

Job :	1	2	3	4	5
Machine A :	5	7	6	9	5
Machine B :	2	1	4	5	3
Machine C :	3	7	5	6	7

Determine the sequence for the jobs that will minimize the total elapsed time.

[(MATH2111.1, MATH2111.5, MATH2111.6)(Apply/IOCQ)]

6 + 6 = 12

9. (a) Listed in the table are the activities and sequencing required in the computerization of a bank branch.

Activity	Description	Predecessor Activity
A	Preparation/Deliberations in the department	–
B	Dialogue with the union	A
C	Discussion/Approval/Sanction of local management	A
D	Customer education	B
E	Preparing specifications for the system	C
F	Selection of staff	B
G	Order/Acquisition of system	D, E
H	Alterations in the branch premises	C
I	Training of staff	F
J	Wiring/Power-supply setup of the branch	H
K	Transition	G, I, J
L	Parallel run	K

Draw the network diagram of the activities associated with the project.

[(MATH2102.1, MATH2102.5, MATH2102.6)(Understand/LOCQ)]

(b) An established company has decided to add a new product to its line. It will buy the product from manufacturing concern, package it, and sell it to a number of distributors that have been selected on a geographical basis. Market research has already indicated the volume expected and the size of sales force required. The steps shown in the following table are to be planned.

Activity	*Description*	*Predecessors*	*Duration* (*days*)
A	Organize sales office	–	6
B	Hire salesmen	*A*	4
C	Train salesmen	*B*	7
D	Select advertising agency	*A*	2
E	Plan advertising campaign	*D*	4
F	Conduct advertising campaign	*E*	10
G	Design package	–	2
H	Setup packaging facilities	*G*	10
I	Package initial stocks	*J*, *H*	6
J	Order stock from manufacturer	–	13
K	Select distributors	*A*	9
L	Sell to distributors	*C*, *K*	3
M	Ship stocks to distributors	*I*, *L*	5

(i) Draw an arrow diagram for this project.
(ii) Indicate the critical path.

[(MATH2102.1, MATH2102.5, MATH2102.6)(Apply/IOCQ)]

5 + 7 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	13.54	55.21	31.25

Course Outcome (CO):

After the completion of the course students will be able to
MATH2204.1 Represent real-world optimization problems by mathematical models and solve them by various techniques.
MATH2204.2 Categorize Transportation and Assignment problems.
MATH2204.3Apply the way in which Game Theoretic Models can be made useful to a variety of real-world scenarios in economics and in other areas.
MATH2204.4.Understand the limitations of simplex method and have realistic approach towards practical problems using Integer Linear Programming Problem.
MATH2204.5. Understand the significance of using PERT and CPM techniques for project management.
MATH2204.6. Solve some specific problems for scheduling jobs on machines.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.