MATHEMATICAL FOUNDATIONS
(MATH 1102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The minimum number of edges in a connected graph with n vertices is equal to
(a) $n(n-1)$ (b) $\frac{n(n-1)}{2}$ (c) n^2 (d) $n-1$.

(ii) A minimally connected graph becomes disconnected when one of the following is removed
(a) one vertex (b) one edge (c) two edges (d) two vertices.

(iii) Corresponding to the adjacency matrix $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ there exists
(a) a connected graph with four vertices
(b) a connected graph with four edges
(c) a disconnected graph with two vertices
(d) a disconnected graph with two components.

(iv) Two balls are drawn successively from a bag containing 6 red and 4 white balls. The probability that at least one of them will be red is
(a) $\frac{78}{90}$ (b) $\frac{48}{90}$ (c) $\frac{30}{90}$ (d) $\frac{12}{90}$.

(v) Three unbiased coins are tossed simultaneously. This is repeated four times. The probability of getting at least one head each time is
(a) $(\frac{1}{8})^4$ (b) $(\frac{2}{8})^4$ (c) $(\frac{3}{8})^4$ (d) $(\frac{7}{8})^4$.

(vi) The mean and variance of a random variable X having binomial distribution are 4 & 2 respectively. Then $P(X=1)=$
(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{1}{32}$.

(vii) The number of internal vertices of a binary tree with 47 vertices is
(a) 21 (b) 22 (c) 23 (d) 24.

(viii) Let X and Y be two random variables such that $Y = a + bX$ where a and b are constants. Then, $Var(Y)$ is

(a) $b^2 Var(X)$ (b) $Var(X)$ (c) $a^2 Var(X)$ (d) $\frac{b}{a}\ Var(X)$.

(ix) If X is normally distributed with zero mean and unit variance, then the expectation of X^2 is

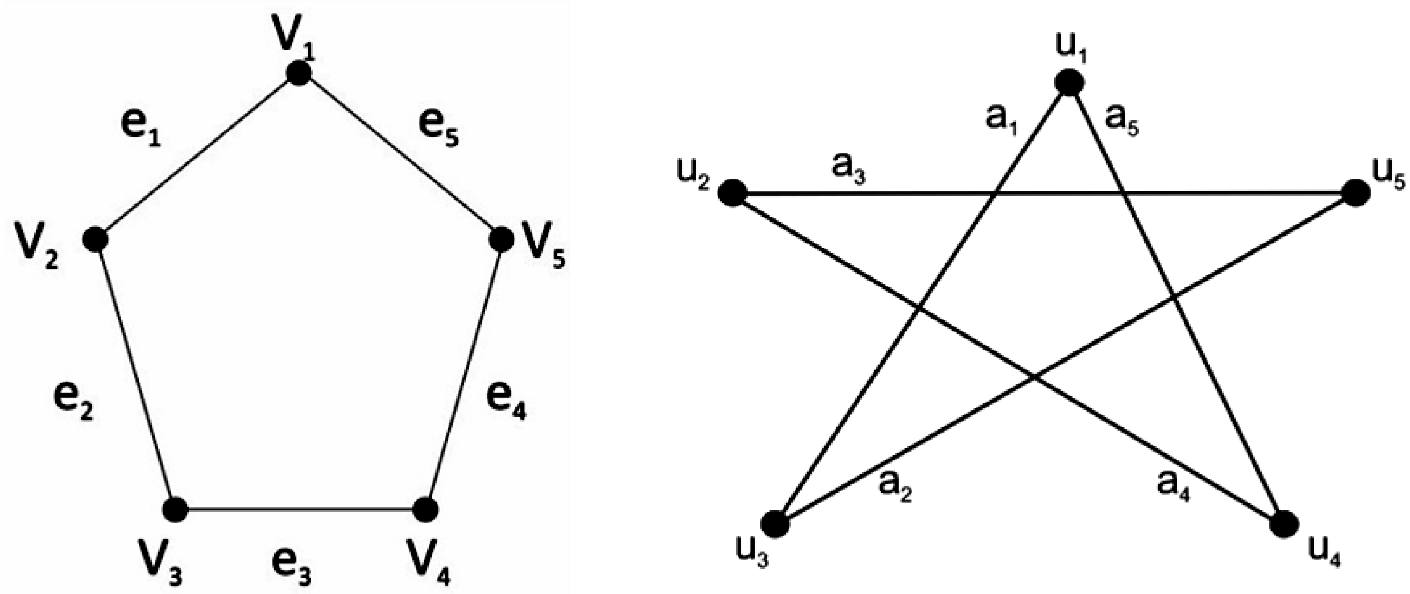
(a) 1 (b) 2 (c) 8 (d) 0.

(x) The generating function of the Fibonacci numbers is

(a) $\frac{1}{1+x+x^2}$ (b) $\frac{1}{1+x-x^2}$ (c) $\frac{1}{1-x-x^2}$ (d) $\frac{1}{1-x+x^2}$.

Group – B

2. (a) Find the incidence matrix of the following two graphs G and G^* and hence examine whether they are isomorphic or not. [(MATH1102.1, MATH1102.3)(Analyze/IOCQ)]



(b) Construct a graph represented by the following incidence matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

[(MATH1102.1, MATH1102.3)(Create/HOCQ)]

7 + 5 = 12

3. (a) Find the minimum and maximum number of edges of a simple graph having 15 vertices and 4 components. [(MATH1102.1, MATH1102.3)(Remember/LOCQ)]

(b) Let T be a tree with 32 edges. On removal of a certain edge from T, two disjoint trees T_1 and T_2 are obtained. If the number of vertices in T_1 is twice the number of vertices in T_2, find the number of edges in T_1 and T_2.

[(MATH1102.1, MATH1102.3)(Understand/LOCQ)]

(c) If G be a graph with n vertices and $(n-1)$ edges, then prove that G either has a pendant vertex or an isolated vertex. [(MATH1102.1, MATH1102.3)(Create/HOCQ)]

4 + 4 + 4 = 12

Group – C

4. (a) A box contains 5 defective and 10 non-defective lamps. Eight lamps are drawn at random in succession without replacement. What is the probability that the 8th lamp is the 5th defective? [(MATH1102.1, MATH1102.2, MATH1102.4)(Evaluate/HOCQ)]

(b) A random variable X has the following probability mass function:

x	0	1	2	3	4	5	6
$P(X = x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

(i) Find the value of k.
(ii) Obtain the distribution function $F(x)$.
(iii) Find $P(3 < X \leq 5)$. [(MATH 1102.1, MATH 1102.2, MATH 1102.4)(Apply/IOCQ)]

6 + 6 = 12

5\. (a) Find the constant k so that the function $f(x) = \begin{cases} kx^2, 0 < x < 3 \\ 0, \quad otherwise \end{cases}$ is a probability density function of the random variable X. Find the distribution function and evaluate $P(1 < X < 2)$. [(MATH1102.1, MATH1102.2, MATH1102.4)(Apply/IOCQ)]

(b) The probabilities of X, Y and Z becoming the principal of a college are respectively 0.3, 0.5 and 0.2. The probabilities that 'Student Aid-Fund' will be introduced in the college if X, Y and Z become principal, are 0.4, 0.6 and 0.1 respectively. Given that 'Student Aid-Fund' has been introduced, find the probability that Y has been appointed as the principal. [(MATH1102.1, MATH1102.2, MATH1102.4)(Evaluate/HOCQ)]

6 + 6 = 12

Group – D

6\. (a) In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts?
[(MATH1102.1, MATH1102.2, MATH1102.4, MATH1102.5)(Remember/LOCQ)]

(b) If the weekly wage of 10,000 workers in a factory follows normal distribution with mean and standard deviation $Rs. 70$ and $Rs. 5$ respectively, find the expected number of workers whose weekly wages are
(i) between $Rs. 66$ and $Rs. 72$;
(ii) less than $Rs. 66$. [(MATH 5101.1, MATH 5101.2, MATH 5101.4)(Apply/IOCQ)]

6 + 6 = 12

7\. (a) Following is a frequency distribution lacking two class frequency. Find them if the mean is 7.74.

value	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13	Total
frequency	32	–	57	–	25	200

[(MATH1102.1, MATH1102.2, MATH1102.4, MATH1102.5)(Apply/IOCQ)]

(b) If x and y are two variables, determine the value of k such that $x + ky$ and $x+\frac{\sigma_x}{\sigma_y}y$ are uncorrelated.
[(MATH1102.1, MATH1102.2, MATH1102.4, MATH1102.5)(Evaluate/HOCQ)]

6 + 6 = 12

Group – E

8\. (a) Let n be a positive integer. If $(n + 1)$ integers not exceeding $2n$ are selected, show that there must be an integer amongst them that is a divisor of one of the other integers. [(MATH1102.1, MATH1102.6)(Create/HOCQ)]

(b) State the Principle of Inclusion-Exclusion for two sets. Use it to solve the following problem. From a group of 10 professors how many ways can a committee of 5 members be formed so that at least one of Professor A and Professor B will be included? [(MATH1102.1, MATH1102.6)(Evaluate/HOCQ)]

6 + 6 = 12

9. (a) During a four-week vacation, a school student will attend at least one computer class each day, but he won't attend more than 40 classes in all during the vacation. Prove that, no matter how he distributes his classes during the four weeks, there is a consecutive span of days during which he will attend exactly 15 classes.
[(MATH1102.1, MATH1102.6)(Evaluate/HOCQ)]

(b) Solve the following recurrence relation by using the method of generating functions.
$a_n - 7a_{n-1} + 10a_{n-2} = 0$ for $n \geq 2, a_0 = 10, a_1 = 41$.
[(MATH1102.1, MATH1102.6)(Create/HOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	14.58	32.29	53.13

Course Outcome (CO):

After the completion of the course students will be able to
MATH1102.1. Understand the mathematical fundamentals which are prerequisites for a variety of courses like data mining, computer security, software engineering, operating systems, machine learning etc.
MATH1102.2. Analyze probability distributions required to quantify phenomenon whose true value is uncertain.
MATH1102.3. Interpret the problems that can be formulated in terms of graphs and trees.
MATH1102.4. Demonstrate the knowledge of probabilistic approaches to solve wide range of engineering problem.
MATH1102.5. Employ statistical methods to make inferences on results obtained from an experiment.
MATH1102.6. Develop the understanding of the mathematical and logical basis to many modern techniques like machine learning, programming language design and concurrency.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.