B.TECH/ECE/4TH SEM/MATH 2202/2021

ADVANCED NUMERICAL METHODS
(MATH 2202)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The eigenvalue of the matrix $A=\begin{bmatrix}4 & -5\\ 2 & -3\end{bmatrix}$ corresponding to the eigenvector $X=\begin{bmatrix}5\\ 2\end{bmatrix}$ is given by:

(a) 1 (b) 2 (c) 3 (d) 4.

(ii) The dominant eigenvalue of the matrix $A=\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{bmatrix}$ is

(a) 2 (b) 1 (c) 0 (d) none.

(iii) For a matrix $A=(a_{ij})_{n\times n}$ with eigenvalue λ, which one of the following is true?

(a) $|\lambda|\le \max\left[\sum_{j=1}^{n}|a_{ij}|\right]$ (b) $|\lambda|\ge \max\left[\sum_{j=1}^{n}|a_{ij}|\right]$

(c) $|\lambda| = \max\left[\sum_{j=1}^{n}|a_{ij}|\right]$ (d) $|\lambda| = \min\left[\sum_{j=1}^{n}|a_{ij}|\right]$.

(iv) $\Delta^2(ab^x)$ is

(a) $a(b-1)b^x$ (b) $a(b-1)^2b^x$ (c) $a(b-1)^2$ (d) a^2b^2x.

(v) Choose the wrong statement from the following:
For the data points $(x_0,y_0),(x_1,y_1),\cdots(x_n,y_n)$, the cubic spline $f(x)$ is such that

(a) $f(x)$ is a quadratic polynomial outside the interval (x_0,x_n)

(b) $f(x)$ is a cubic polynomial in each of the subintervals

(c) $f'(x)$ is continuous at each given data point

(d) $f''(x_0)=f''(x_n)=0$.

(vi) $[x, x_0, x_1] =$

(a) $[x_0, x_1, x_2] + (x - x_0)[x, x_0, x_1, x_2]$ (b) $\frac{[x,x_1]-[x,x_0]}{x_0-x_1}$

(c) $[x_0, x_1, x_2] + (x - x_2)[x, x_0, x_1, x_2]$ (d) $\frac{[x,x_1]-[x_1,x_0]}{x-x_1}$.

(vii) For a system of 3 linear equations with 3 unknowns, the maximum number of addition and subtraction in Gauss-elimination method is

(a) 9 (b) 10 (c) 11 (d) 7.

(viii) Suppose $A = GG^T$ is the Cholesky factorization of a symmetric positive definite matrix A, where G is

(a) a lower triangular matrix (b) an upper triangular matrix

(c) a scalar matrix (d) an orthogonal matrix.

(ix) Out of the following search algorithms, which one has the fastest rate of convergence:

(a) Interval halving method (b) Dichotomous search

(c) Fibonacci search (d) Golden section search.

(x) A real $n \times n$ symmetric matrix A is such that $X^T AX \geq 0$ for some non-zero vector X and $X^T AX \leq 0$ for some non-zero vector X. Then A will be

(a) positive definite (b) negative definite

(c) indefinite (d) positive semi definite.

Group – B

2. (a) Solve the system of linear equations using Gauss-elimination method with partial pivoting correct upto 3 places of decimal:

$$x_1 + 10x_2 - x_3 = 3$$
$$2x_1 + 3x_2 + 20x_3 = 7$$
$$10x_1 - x_2 + 2x_3 = 4$$

(b) (i) What is condition number of a system of linear equations? Using condition number, derive the upper bound in the relative error of the solution of the system of linear equations $Ax = b$ if the vector b is mistakenly considered as $\hat{b}(= b + r)$.

(ii) Assuming the $\| \cdot \|_\infty$ norm, compute the condition number of the system of linear equations $Ax = b$, where $A = \begin{bmatrix} 1.01 & 0.99 \\ 0.99 & 1.01 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$.

6 + (4 + 2) = 12

3. Compute the Cholesky factorization of the matrix $A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 4 \end{bmatrix}$.

Hence solve the following system:

$$3x_1 - x_2 = 5$$
$$-x_1 + x_2 + x_3 = 0$$
$$x_2 + 4x_3 = 15$$

(6 + 6) = 12

Group – C

4. Apply inverse power method (with scaling) on the following matrix to obtain the eigenvalue with the smallest magnitude and the corresponding eigenvector:

$$A=\begin{bmatrix}3 & 0 & 0\\ 1 & -1 & 0\\ 0 & 2 & 8\end{bmatrix}$$

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5. Find the singular values of $A=\begin{bmatrix}0 & 1 & 1\\ \sqrt{2} & 2 & 0\\ 0 & 1 & 1\end{bmatrix}$ and find the Singular Value Decomposition (SVD) of A.

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Group – D

6. (a) Obtain the cubic spline approximation for the function defined by the data:

x:	1	2	3	4
$f(x)$:	1	2	5	11

with $M_0=0,\ \ M_3=0$, where $M_{x_i}=f''(x_i)$. Hence find an estimate of $f(1.5)$.

(b) Given the values

x:	1.0	1.1	1.3	1.5	1.6
$f(x)$:	0.3639	0.3258	0.2612	0.2095	0.1876

Evaluate $f(1.25)$ using Newton's divided difference formula.

8 + 4 = 12

7. (a) Find the number of men getting wages between Rs.10 and Rs.15 from the following data using Newton's forward interpolation formula:

Wages in Rs.	$0-10$	$10-20$	$20-30$	$30-40$
Frequency	9	30	35	42

(b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using

(i) Simpson's $\frac{3}{8}$ rule.

(ii) Weddel's rule

Dividing the range in six equal subintervals.

6 + (3 + 3) = 12

Group – E

8. Use Dichotomous Search algorithm tominimize the function $f(x)=2-4x+e^x$ in the interval $[0.5, 2.5]$taking a tolerance value less than 0.3. Consider $\varepsilon=0.002$.

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9. Derive the value of Golden ratio and use the Golden Section Search technique to maximize the function $f(x) = -3x^2 + 21.6x + 1$ in the interval [0,25] taking tolerance to be less than 1.0.

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Department & Section	Submission Link
ECE-A	https://classroom.google.com/c/MzExMjI0Njk5MzYy/a/MzczMTE4NjQ3MzEw/details
ECE-B	https://classroom.google.com/c/MzExMjI0Njk5NDAw/a/MzczMTIwNzk0OTE4/details
ECE-C	https://classroom.google.com/c/MzMxMDc5NjU1NzQ0/a/MzczMTM1NTY0Mjgx/details