MCA /1ST SEM/MCAP 1104/2018
DISCRETE MATHEMATICS
(MCAP 1104)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A debating team consists of 3 boys and 2 girls. Find the number of ways they can sit in a row.
(a) 120 (b) 24 (c) 720 (d)12.

(ii) Which of the following pair is not congruent modulo 7?
(a)10, 24 (b) 25, 56 (c) -31, 11 (d) -64, -15 .

(iii) How many onto (or surjective) functions are there from an n-element (n => 2) set to a 2-element set?
(a) 2^n (b) $2^n - 1$ (c) $2^n - 2$ (d) $2(2^n - 2)$.

(iv) The relation {(1,2)(1,3)(3,1)(1,1)(3,3)(3,2)(1,4)(4,2)(3,4)} is
(a)Rellexive (b)Trasitive
(c) Symmetric (d) Antysymmetric.

(v) Push down machine represents
(a) Type 0 Grammar (b) Type 1 Grammar
(c) Type 2 Grammar (d) Type 3 Grammar

(vi) $((P\rightarrow(Q\vee R))\wedge(\sim Q))\rightarrow(P\rightarrow R)$
(a) is a Contradiction (b) is a tautology
(c) neither tautology nor contradiction (d) none of these.

(vii) Consider the recurrence relation $a_k = 6a_{k-1} - 9a_{k-2}$ with initial conditions $a_0 = 0$ and $a_1 = 2$. Which of the following is an explicit solution to this recurrence relation, provided the constants A and B are chosen correctly?
(a) $a_n = A3^n + B3^n$ (b) $a_n = A3^n + B(-3)^n$
(c) $a_n = A3^n + nB^3n$ (d) $a_n = A(-3)^n + nB(-3)^n$

(viii) Find the generating function for the following numeric function - <0,1,2,3,4,……>

(a) $\frac{x}{(1-x)^2}$ (b) $\frac{x}{(1+x)^2}$ (c) $\frac{x}{(1-x)}$ (d) $\frac{x}{(1+x)}$.

(ix) If every edge of a graph has equal weight then
(a) minimal spanning tree is not obtained
(b) minimal spanning tree is unique
(c) every spanning tree is minimal spanning tree
(d) every tree will be spanning tree.

(x) If the letters of the word LATE are arranged in the order of dictionary, what will be the rank of the word LATE.
(a)13 (b)14 (c)15 (d)12.

Group – B

2.(a) What is equivalence relationship?
A relation R defined on the set Z by "aRb iff ab>0 for all a,b ∈ Z". Examine whether R is (i) reflexive (ii) symmetric (iii) transitive.

(b) Let P={1,2,3,4} be a non empty set and ≤ be the relationship. Show that {P, ≤} be a poset and draw the corresponding Hasse diagram.

(c) Show that the function f: $\mathbb{R}\rightarrow\mathbb{R}$ defined by

$$f(x)=\begin{cases} x+6, & x<-3 \\ \sqrt{3-2x}, & -3\le x\le 1 \\ 4x-3, & x>1 \end{cases}$$

is bijective.

(1 + 3) + (3 + 1) + 4 = 12

3.(a) (i) For any 3 sets A,B,C prove that
(A∩B) U(B∩C) U(C∩A) = (AUB) ∩(BUC) ∩((CUA).
(ii) Consider a set of integers from 1 to 250. Find how many of these numbers are divisible by 3 or 5 or 7.Also indicate how many of these are divisible by 3 or 7 but not by 5 and divisible by 3 or 5.

(b) Find the disjunctive normal form $(\sim P\rightarrow R)\wedge(P\leftrightarrow Q)$.

(c) Prove that the following tautology.
$((P\rightarrow Q)\wedge(R\rightarrow S)\wedge(P\vee R))\rightarrow(Q\vee S)$

(3 ×2) + 3 + 3 = 12

Group – C

4 (a) Find the solution of the recurrence relation
$a_{n+2} - a_{n+1} - 2a_n = 0$, $n \ge 0$, satisfying the initial condition $a_0=3$ and $a_1=3$

(b) Find the general solution of the recurrence relation
$a_{n+2} - 5a_{n+1} + 6a_n = 2^{n0}$, $n \ge 0$

(c) State and prove the Pigeonhole principle.

4+ 4 + 4 = 12

5. (a) Find the coefficient of X^{18} from the expansion $(X^3 + X^4 + \ldots\ldots\ldots)^3$

(b) Five points are given on one of two parallel straight lines and 10 points are given on the other straight line. How many triangles can be formed by taking these points as the vertices of a triangle?

(c) How many different 4-letter words can be made out of the letters of the word "EXAMINATION"?

4+ 4 + 4 = 12

Group – D

6. (a) Draw the expression tree for the arithmetic expression (((a+b*c)*d-e)/(f+g))+h*(i*j-k) using the precedence rule of the binary operators.

(b) Prove that, a graph is a tree if and only if it is minimally connected.

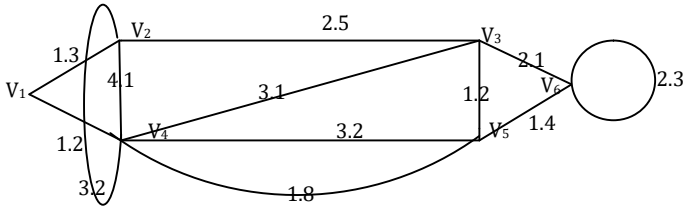
(c) Prove that a graph with n vertices and k components can have a maximum of $\frac{(n-k)(n-k+1)}{2}$ number of edges. From this find out an expression for maximum number of edges in a completely connected graph with n vertices.

4 + 3 + 5= 12

7. (a) i) Define tree and binary tree . Prove that a tree with *n* vertices always has $(n-1)$ edges.

ii) What do you mean by a spanning-tree of a connected graph G. What is minimum spanning-tree ?

(b) Applying Prim's algorithm, find the minimum spanning tree for the following weighted connected graph.



(1 + 4) + (1 + 2) + 4=12

Group – E

8. (a) Consider the following NDFA

q	δ(q,0)	δ(q,1)
a	{a,b,c,d,e}	{d,e}
b	{c}	{e}
c	φ	{b}
d	{e}	φ
e	Φ	φ

Evaluate the corresponding DFA and draw the diagram. Minimize the state of the DFA if possible.

(b) (i) What is the fundamental difference between finite automata and Mealy machine.

(ii) Convert the following Moore machine into it's equivalent Mealy machine . Clearly mention the steps involved.

Present State	Next State		Output
	a=0	a=1	
→a	d	b	1
b	a	d	0
c	c	c	0
d	b	a	1

6 + (2 + 4) =12

9. (a) What are regular expression and regular language? Show that the language $\{0n10n : n \geq 1\}$ is not a regular language.

(b) Define context sensitive language with example.

(c) Consider the grammar G with V={S, A,B}, ∑={a, b} and
P= {S-> aAS, S->a, A->SbA, A->ba}
Construct the derivation tree for the word aabbaa. Also identify the type of grammar.

(2 + 4) +(2 + 3 + 1) = 12