

**NUMERICAL AND STATISTICAL TECHNIQUES
(MCAP 1103)**

Time Allotted : 3 hrs

Full Marks : 70

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.*

Candidates are required to give answer in their own words as far as practicable.

**Group - A
(Multiple Choice Type Questions)**

1. Choose the correct alternative for the following: **10 × 1 = 10**
- (i) If $P(\bar{A} \cup \bar{B}) = 5/6$, $P(A) = 1/2$ and $P(\bar{B}) = 2/3$ then the events A and B are
 (a) mutually exclusive (b) mutually independent
 (c) mutually exhaustive (d) none of the above.
- (ii) For what value of a is $f(x) = a(1/2)^x$, $x = 0, 1, 2, \dots$, a probability mass function of a random variable X ?
 (a) $1/4$ (b) $2/7$ (c) $1/2$ (d) 1.
- (iii) X and Y are both negative random variables, mutually independent, $E(XY) = 6$ and $|E(X)| = 2$. Then the value of $E(Y)$ is
 (a) 3 (b) -3 (c) 12 (d) ± 3 .
- (iv) If T_1 and T_2 are statistics with $E(T_1) = 2\theta_1 + 3\theta_2$ and $E(T_2) = \theta_1 + \theta_2$, then the unbiased estimator of θ_1 is
 (a) $3T_2 - T_1$ (b) $3T_2 + 2T_1$ (c) $2T_2 - 3T_1$ (d) $3T_2T_1$.
- (v) Which of the following is Type II error?
 (a) The error of accepting H_0 when H_0 is false
 (b) The error of accepting H_0 when H_0 is true
 (c) The error of rejecting H_0 when H_0 is true
 (d) The error of rejecting H_0 when H_0 is false.
- (vi) The number 3.4506531 when rounded off to 4 places of decimal becomes
 (a) 3.4506 (b) 3.4507 (c) 3.451 (d) 3.4505.
- (vii) Newton-Raphson method fails when
 (a) $f'(x)$ is negative (b) $f'(x)$ is too large
 (c) $f'(x)$ is zero (d) never fails.

- (viii) Gauss-Seidel iteration converges only if the coefficient matrix is
 (a) upper triangular (b) lower triangular
 (c) diagonally dominant (d) banded matrix.
- (ix) The order of convergence in regula falsi method is
 (a) 1.235 (b) 3.141 (c) 1.618 (d) 2.792.
- (x) In general, the ratio of the truncation error to that of round off error is
 (a) 2:1 (b) 1:1 (c) 1:2 (d) 1:3.

Group - B

2. (a) For any two events A_1 and A_2 , prove that
 (i) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$
 (ii) $P(A_1 \cap A_2) \geq P(A_1) + P(A_2) - 1$
- (b) Two newspapers X and Y are published in a certain city. It is estimated from a survey that 16% read X , 14% read Y and 5% read both. Find the probabilities that a randomly selected person (i) does not read any newspaper (ii) reads only Y .

(5 + 1) + (3 + 3) = 12

3. (a) A random variable X has the following probability density function:

$$f(x) = \begin{cases} k, & -2 < x < 2 \\ 0, & \text{otherwise,} \end{cases}$$
 (i) Determine the constant k
 (ii) What is the value of $P(|X| > 1)$?
- (b) Three balls are drawn at random from a bag containing 5 white and 3 black balls. If X denotes the number of white balls drawn, find the probability distribution of X , $E(X)$ and $\text{Var}(X)$.

(3 + 3) + (3 + 1 + 2) = 12

Group - C

4. (a) If X_1, X_2, X_3, X_4, X_5 and X_6 are independent simple random sample from a normal population with unknown variance σ^2 , find k so that $k[(X_1 - X_2)^2 + (X_3 - X_4)^2 + (X_5 - X_6)^2]$ is an unbiased estimator of σ^2 .
- (b) A random sample of 200 apples was drawn from a very large-sized consignment and 25 of them were found to be bad. Find the unbiased estimate of variance of sample proportion of bad apples.

6 + 6 = 12

5. (a) Distinguish between the following, in context of hypothesis testing: Type-I error and Type-III error, one-sided test and two-sided test.
- (b) The standard deviation of 100 W light bulbs whose life span follows a normal distribution is given to be 120 hours and a half life of same is guaranteed to be 700 hours of under warranty. 100 bulbs are selected at random from the lot whose half-life is determined to be 650 hours. If the significance level is 0.01, then should the lot be rejected or not for not complying with the guarantee?
- (4 + 2) + 6 = 12**

Group - D

6. (a) Find the interpolation polynomial of degree 5 which supports the following data points using Newton's divided-difference formula:

x	0	1	2	3	4	5
y	4	3	-4	-23	-60	-121

Now find the value of $f(2.5)$

- (b) Prove that

$$(i) \Delta^n \left(\frac{1}{x} \right) = (-1)^n \frac{n!h^n}{x(x+h)(x+2h)\dots(x+nh)}$$

$$(ii) E = e^{hD}$$

(5 + 1) + (4 + 2) = 12

7. (a) From the following table find the polynomial $f(x)$ by Newton's divided difference interpolation formula.

x	-1	0	3	6	7
$f(x)$	3	-6	39	822	1611

- (b) A tank is discharging water through an orifice at a depth of x metre below the surface of the water whose area is A m². Following are the values of x for the corresponding values of A .

A	1.257	1.39	1.52	1.65	1.809	1.962	2.123	2.295	2.462	2.650	2.827
x	1.5	1.65	1.8	1.95	2.1	2.25	2.4	2.55	2.7	2.85	3

Using the formula $(0.018)T = \int_{1.5}^{3.0} \frac{A}{\sqrt{x}} dx$, calculate T (the time in seconds) for the level of the water to drop from 3.0 mt to 1.5 mt above the orifice.

6 + 6 = 12

Group - E

8. (a) Find the root of the equation $xe^x = \cos x$ in the interval (0,1) using Regula Falsi method correct to four decimal places.

- (b) If $\frac{dy}{dx} = \frac{y-x}{y+x}$, find the value of y at $x = 0.1$ and $x = 0.2$ using Runge-Kutta method (fourth order). Given that $y(0) = 1$.

6 + 6 = 12

9. (a) Find a real root of the equation $x^3 + x^2 - 1 = 0$ on the interval [0,1] with an accuracy of 10^{-4} by Bisection method.

- (b) Applying modified Euler's method, solve the following:

$$\frac{dy}{dx} = e^x + xy, y(0) = 0$$

Calculate $y(0.1)$ and $y(0.2)$.

4 + (4 + 4) = 12