

7. (a) Determine the z-transform and the Region of Convergence for the following signal

$$x[n] = -\left(\frac{1}{4}\right)^n u[-n-1] - \left(-\frac{1}{2}\right)^n u[-n-1]$$

- (b) (i) Compare analog and digital filters
 (ii) Given impulse response of a system $h(n) = \frac{1}{2}[\delta(n) + \delta(n-1)]$. Identify the type of filter and justify your answer.

4 + (3 + 5) = 12

Group - E

8. (a) (i) What is meant by autocorrelation function?
 (ii) Explain the relation between autocorrelation function and energy spectral density function of a signal.
- (b) A continuous random variable has the probability density function defined by $f(x) = \frac{2}{9(x-1)}$ for $1 < x < 4$
 $= 0$, otherwise
 Determine the probability distribution function, the mean and variance of the random variable.

5 + 7 = 12

9. Write short notes on any **three** of the following

- BIBO stability
- Region of Convergence
- White Gaussian Noise
- Thermal Noise
- Causal and anti-causal system
- Modulation

(4 × 3) = 12

**SIGNALS AND SYSTEMS
(ECEN 2103)**

Time Allotted : 3 hrs

Full Marks : 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

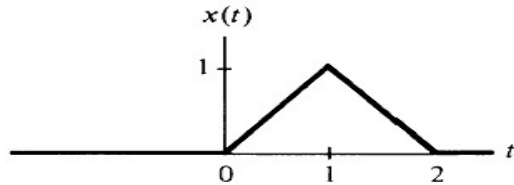
**Group - A
(Multiple Choice Type Questions)**

1. Choose the correct alternative for the following: **10 × 1 = 10**
- A system which is linear is said to obey the rules of
 (a) scaling (b) additivity
 (c) both scaling and additivity (d) homogeneity.
 - The harmonics of the Fourier series of an even signal consists of
 (a) sine terms only
 (b) cosine terms only
 (c) both sine and cosine terms
 (d) neither sine nor cosine terms.
 - The step function $u(t)$ is integral of _____ with respect to time t .
 (a) ramp function (b) impulse function
 (c) sinusoidal function (d) exponential function.
 - Which of the following is correct regarding impulse signal?
 (a) $x[n]\delta[n] = x[0]\delta[n]$ (b) $x[n]\delta[n] = \delta[n]$
 (c) $x[n]\delta[n] = x[n]$ (d) $x[n]\delta[n] = x[0]$.
 - Aliasing occurs when the signal is
 (a) over-sampled (b) under-sampled
 (c) critically sampled (d) not sampled.
 - In Laplace transform multiplication by e^{-at} in time domain becomes
 (a) translation by a in s domain
 (b) translation by $(-a)$ in s domain
 (c) multiplication by e^{-as} in s domain
 (d) none of the above.

- (vii) The discrete-time signal $x[n] = (-1)^n$ is periodic with a fundamental period of
 (a) 6 (b) 4 (c) 2 (d) 1.
- (viii) If the maximum frequency of a signal $x(t)$ generated by a low pass filter is 5 kHz then, the maximum frequency in $x(t)\cos(2000\pi t)$, in kHz, is
 (a) 5kHz (b) 1kHz (c) 4kHz (d) 6kHz.
- (ix) A system whose output depends on future inputs is a
 (a) static system (b) dynamic system
 (c) non-causal system (d) both (b) and (c).
- (x) Fourier transform of continuous time aperiodic signal is
 (a) continuous and aperiodic (b) discrete and aperiodic
 (c) continuous and periodic (d) discrete and periodic.

Group - B

2. (a) Find the even and odd components of the following signal



- (b) Given $x(t) = 3\sin(10\pi t) + 4\cos(20\pi t)$. Examine whether the signal is periodic or not. If yes, determine the period of $x(t)$.
- (c) Sketch the following signal and determine whether it is an energy or power signal. (i) $u(t) + u(t-2)$ (ii) $e^{-5t}u(t)$

3 + 3 + 6 = 12

3. (a) An LTI system is defined by the differential equation

$$\frac{d^2 y(t)}{dt^2} + 4y(t) = \frac{3dx(t)}{dt}.$$

Find (i) Natural response if the initial conditions are $y(0) = -1$; $\frac{dy(0)}{dt} = 0$

(ii) Forced response if the input is $x(t) = e^{-2t}u(t)$

- (b) Determine the linearity of $\frac{dy(t)}{dt} + 2y(t) = x(t)\frac{dx(t)}{dt}$

- (c) Determine whether the system $y(t) = e^{x(t)}$ is time-variant or time-invariant.

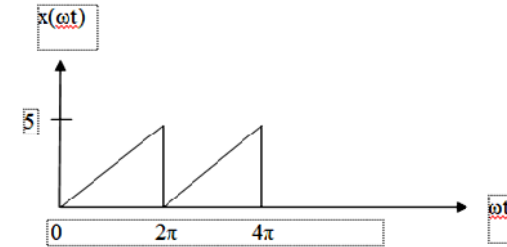
7 + 3 + 2 = 12**Group - C**

4. (a) (i) Explain the Dirichlet conditions as applicable to Fourier transform.
 (ii) Compute the Fourier transform of the signum function.
- (b) State and prove Duality Property of Fourier transform. Hence find the Fourier transform of $\frac{1}{t}$
5. (a) Using Laplace transform, find the impulse response of an LTI system described by the differential equation

$$\frac{d^2 y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

(2 + 4) + (3 + 3) = 12

- (b) Find the trigonometric Fourier series for the following periodic function



- (c) Compute the discrete-time Fourier transform of $x[n] = (1/4)^n u[n]$

3 + 6 + 3 = 12**Group - D**

6. (a) Determine the Nyquist rate for a continuous-time signal $x(t) = 6\cos 50\pi t + 20\sin 300\pi t - 10\cos 100\pi t$.
- (b) Explain aliasing with proper waveforms and how it can be eliminated.
- (c) Distinguish between Natural sampling and Flat top sampling.

3 + (3+2) + 4 = 12