B.TECH/CHE/5TH SEM/CHEN 3104/2018

NUMERICAL METHODS OF ANALYSIS

(CHEN 3104)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The number 86.538 is chopped to the 2nd decimal place. The percent true error due to chopping is
(a) 0.008 (b) 0.00009 (c) 0.0092 (d) 0.008.

(ii) The real roots of $\begin{bmatrix} 1 & 1 \\ 1 & 8 \end{bmatrix}$ are
(a) imaginary (b) real and equal
(c) real and unequal (d) zero.

(iii) The norm of a vector ||x|| is equal to _____, where 'a' is a matrix of dimension n
(a) $\max\{S_i\}$, where $S_i = \sum_j a_{ij}$ (b) $\min\{S_i\}$, where $S_i = \sum_j a_{ij}$
(c) $\max\{S_i\}$, where $S_i = \sum_j |a_{ij}|$ (d) $\min\{S_i\}$, where $S_i = \sum_j |a_{ij}|$

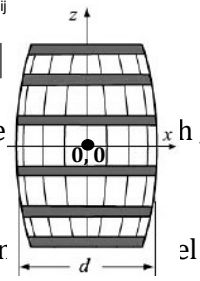
(iv) Inverse of a matrix can be ___ h _____ algorithm
(a) Gauss Siedel (b) Gauss Jordon
(c) Gaussian Elimination (d) Jacobi.

(v) The condition for applyi ___ el iterative method, the matrix must be
(a) diagonally dominant (b) unit matrix
(c) upper triangular matrix (d) lower triangular matrix.

(vi) Newton-Raphson method will always converge to a solution for f(x)=0 on the interval a≤x≤b if certain conditions are met. Which of the following is not one of these conditions?
(a) f is continuous on the interval a≤x≤b
(b) f(a) and f(b) have opposite signs
(c) f''(x) does not change sign on the interval a≤x≤b
(d) f'(x)=0 on the interval a≤x≤b.

Group – B

2. (a) To estimate the surface area of a drum, the diameter of the drum is measured at various points on the drum.



z (inch)	-12	-6	0	6	12
d (inch)	0	3.2	4.4	5.6	6

Find the radius of the drum at z = -8.
What kind of interpolation did you use and why?

(b) The surface area of the drum (shown in Q. 2(a)) can be evaluated by integrating $S = \int_a^b 2\pi r dr$. Evaluate the surface area by applying any of the numerical integration tools using the given data in Q. 2(a).

6 + 6 = 12

3. (a) Water solubility in jet fuel, W_S, as a function of temperature, T, has been measured and the following experimental data reported.

T (°C)	-40	-20	0	20	40
W_S	0.0012	0.002	0.0032	0.006	0.0118

Use Newton's interpolation to obtain values at 5°C.

(b) If a Lagrange polynomial were to be fitted to above data, evaluate the coefficients and state the order of polynomial used.

6 + 6 = 12

Group – C

4. (a)

In the above scheme the separation of A, B and C has been done with the mentioned mass fraction (w). The feed flow rate is given as 10 kg/min. Find out the product flow rates (F1, F2 and F3) using Gaussian Elimination method.

(b) For the mentioned problem in 4.(a) comment on the effect of the feed components' mass fraction on the product quality.

8 + 4 = 12

5. (a) You have a spherical storage tank containing oil. The tank has a diameter of 6 m. You are asked to calculate the height 'h' to which a dipstick 8 m long would be wet with oil when immersed in the tank when it contains 10 m^3 of oil.

The equation that gives the height (h) of the liquid in the spherical tank for the given volume and radius is given by $f(h) = h^3 - 9h^2 + 3.8197 = 0$.

Use the bisection method of finding roots of equations to find the height (h) to which the dipstick is wet with oil. Show maximum three iterations if not converged.

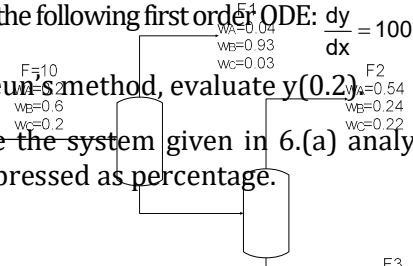
(b) Which method will provide faster convergence – "secant method" or "bisection method"? Explain with proper reasoning.

9 + 3 = 12

Group – D

6. (a) Consider the following first order ODE: $\frac{dy}{dx} = 1000x\left(\frac{x^2}{y}\right)$ with initial value y(0) = 2.

Using Heun's method, evaluate y(0.2).



(b) Integrate the system given in 6.(a) analytically and evaluate the true error expressed as percentage.

7 + 5 = 12

7. (a) A chemical compound decays over time when exposed to air and can be modelled using the equation given below:

$$\frac{dn}{dt} = -0.8n^{3/2} + 10n_0(1 - e^{-3t})$$

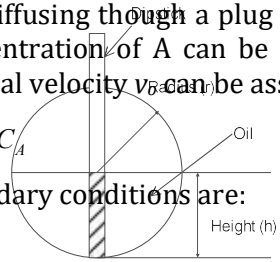
Solve the equation using 2nd order RK method to find the concentration *n* as a function of time using a time step of 0.2 s. The initial concentration n_o is 2000.

(b) Use Taylor series to linearize e^{-3t} as given in the above equation of Q. 7(a) till the 1st order term. Solve the resulting equation using Euler's method. Compare with values obtained in 7(a).

6 + 6 = 12

Group – E

8. (a) Gas species A is diffusing through a plug flow reactor from x = 0 to x = 10 cm. The concentration of A can be modelled using the following PDE. The superficial velocity v_o can be assumed to be 1 cm/s.



$$\frac{\partial C_A}{\partial t} = 0.5\frac{\partial^2 C_A}{\partial x^2} - v_o C_A$$

Appropriate boundary conditions are:

$C_A(0,t) = 1$

$C_A(10,t) = 0$

$C_A(x,0) = 0$ for all $0 < x < 1$

Setup the numerical form of the differential equation in form $\underline{\underline{A}}\,\underline{y} = \underline{B}$ using an implicit scheme. Clearly write out $\underline{\underline{A}}$ and $\underline{B}$.

(b) Choose two intermediate grid points for the above Q.8(a) along positive x axis in addition to zero superficial velocity. Find out C_A for all grid points at t = 0.1 s, taking step size h = 0.1 s.

6 + 6 = 12

9. (a) The following differential equation is to be solved over a domain, 0<x<0.5 and 0<y<0.8, subject to boundary conditions given below.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = 0.5x$$

$x = 0, \textit{for all } y, u = 1$

$x = 0.5, \textit{for all } y, \frac{du}{dx} = u$

$y = 0, \textit{for all } x, u = 0$

$y = 0.8, \textit{for all } x, u = 0$

Write out the numerical form of the difference equation with the domain divided into 3 interior grid points in the x and y directions.

(b) Setup the system in the form $\underline{\underline{A}}\,\underline{y} = \underline{B}$ taking into account the boundary condition.

5 + 7 = 12