B.TECH/ME/5TH SEM/MECH 3142/2018
FINITE ELEMENT METHOD
(MECH 3142)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Stiffness of a bar under axial loading having length 'L', cross-sectional area 'A' and Modulus of rigidity 'E' is

(a) $\frac{AE}{3L}$ (b) $\frac{AE}{L}$ (c) $\frac{E}{AL}$ (d) $\frac{AE}{L^2}$.

(ii) Stiffness matrix of a two dimensional BEAM element is a:

(a) 3 × 3 matrix (b) 4 × 4 Matrix
(c) 2 × 2 matrix (d) Single column matrix.

(iii) Which one among the following is correct regarding stiffness matrix of a two dimensional BEAM element?

(a) $\frac{AE}{3L}[4\times 4]$ (b) $\frac{AE}{3L}[4\times 4]$ (c) $\frac{EI}{L^3}[4\times 4]$ (d) $\frac{AE}{L^2}[4\times 4]$.

All the symbols and notations carry their usual meaning.

(iv) The coefficient in stress-strain relation for a linear, elastic, isotropic material under plane strain condition is given by-

(a) $\frac{E}{(1+v)}\begin{bmatrix} 1-v & 0 & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{1-2v}{2} \end{bmatrix}$ (b) $\frac{E}{(1+v)(1-2v)}\begin{bmatrix} 1-v & 0 & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{1-2v}{2} \end{bmatrix}$

(c) $\frac{E}{(1-2v)}\begin{bmatrix} 1-v & 0 & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{1-2v}{2} \end{bmatrix}$ (d) $\frac{E}{(1+v)(1-2v)}\begin{bmatrix} v & 0 & 0 \\ v & v & 0 \\ 0 & 0 & \frac{1-2v}{2} \end{bmatrix}$

(v) CST element process

(a) Constant filed variable throughout the element
(b) Strain is constant throughout the element
(c) Variation of the field variable is quadratic throughout the element
(d) Variation of the field variable is cubic throughout the element.

(vi) For a Four-Noded Rectangular element having length 'l' and width 'w', if one of the shape functions in user-coordinate (x, y) is expressed as $N_1 = \frac{x}{l}\left(1-\frac{y}{w}\right)$ then the corresponding shape function in normalized coordinate (ξ, η) will be?

(a) $\frac{1}{4}(1-\xi)(1+\eta)$ (b) $\frac{1}{4}(1-\xi)(1-\eta)$

(c) $\frac{1}{2}(1-\xi)(1-\eta)$ (d) $\frac{1}{4}(1-\xi)$.

(vii) The weight factor W_i for a single point integration by Gauss Quadrature formula is

(a) 4 (b) 3 (c) 1 (d) 2.

(viii) For two-point numerical integration by Gauss Quadrature formula, the locations of sampling point (x_i) are

(a) $\left(+\frac{1}{\sqrt{5}},-\frac{1}{\sqrt{5}}\right)$ (b) $\left(+\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{5}}\right)$

(c) $\left(+\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}}\right)$ (d) $\left(+\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$.

(ix) The sequence of the numerical simulation in any FEA software is

(a) Pre-processing → Solution → Post-processing
(b) Post-processing → Solution →Pre-processing
(c) Pre-processing → Post-processing → Solution
(d) Solution→ Pre-processing→Post-processing.

(x) Which one of the following software is a FEA dedicated software?

(a) CATIA (b) CRO Parametric
(c) ANSYS (d) UG NX.

Group – B

2. Use Galarkin's method of weighted residuals to obtain an approximate solution of differential equation.

$$\frac{d^2y}{dx^2} - 10x^2 = 5 \qquad 0 \le x \le 1$$

With boundary conditions y(0) =y(1)=0. (Use two weighting functions)
Also compare the approximate solution with the exact solution of the differential equation.

6 + 6 = 12

3. Using Rayleigh- Ritz method derive the expression for vertical displacement 'v' of a cantilever beam subjected to uniformly distributed load q_0 as shown in Figure 1 below as a function of domain variable 'x'. Consider as approximate solution consisting of two terms.

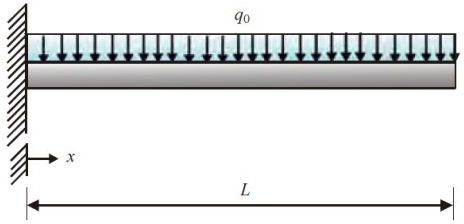

Figure 1

Consider here Modulus of elasticity of the beam is 'E' and Moment of Inertia of the beam cross-section is 'I'.

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Group – C

4. Two trolleys are connected by the arrangement of springs shown in Figure 2 below. (a) Determine the complete set of equilibrium equations for the system in the form [K]{δ}= {F} in global sense. (b) If k=5 N/mm, F1 = 60 N and F2 = 90 N, compute the displacement of each trolley and the force in each spring.

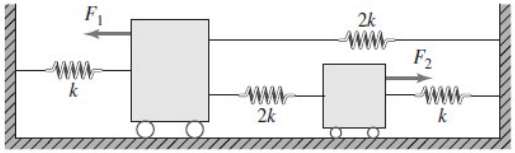

Figure 2

5 + 7 = 12

5. Consider the beam shown in Figure 3 below. The beam has been loaded as shown in figure and is made of a material having modulus of elasticity E = 190 GPa. The cross-sectional area of the beam is rectangular and having dimensions 20mm width and 30mm height. Consider two beam elements. Assemble element equations. Incorporate the boundary conditions. Form the final equations (to be solved) in matrix form (Solution is not required).

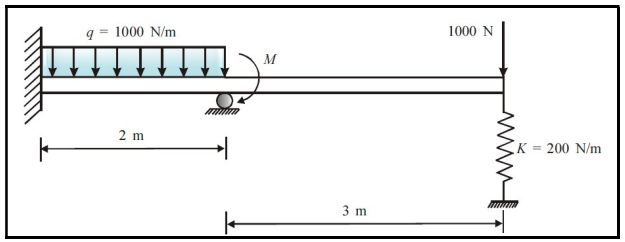

Figure 3

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Group – D

6. Figure 4 shows a two-member plane truss supported by a linearly elastic spring. The truss members are of a solid circular cross section having d = 20 mm and E = 80 GPa. The linear spring has stiffness constant 70 N/mm. Now determine global displacement of the unconstrained node. Also calculate reaction forces at constrained nodes and stresses in bars.

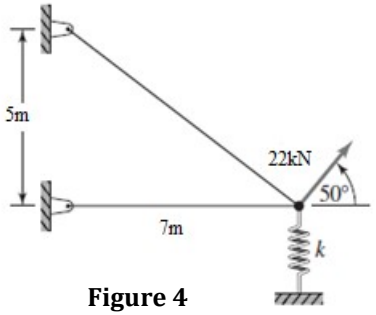

Figure 4

8 + 4 = 12

7. (a) Figure 5 shows a quadrilateral element in global coordinates. Show that the mapping correctly describes the line connecting nodes 2 and 3 and determine the (x, y) coordinates corresponding to (r,s) = (3.5, 1.5). Map the line r = 0 in the parent element to the physical element. Plot the mapping on a scaled drawing of the quadrilateral element

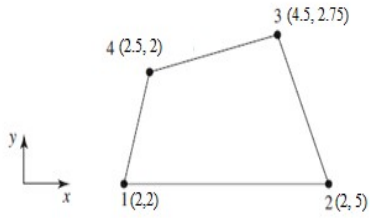

Figure 5

(b) Evaluate the following integral using one-point and two-point Gaussian quadrature. Compare each result with the corresponding analytical solution

$$\int_2^6 (x^2 + 5x + 3)\, dx$$

6 + 6 = 12

Group – E

8. Describe in detail about 'Pre-Processing', procedures in FEM.

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9. Describe in detail about classification of elements on the basis of topology used in a FEA software.

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