MCA/2ND SEM/MCAP 1204/2018

OPTIMIZATION TECHNIQUES
(MCAP 1204)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

   (i) Each constraint in an LP model is expressed as an
   (a) inequality with ≥ sign (b) inequality with ≤ sign
   (c) equation with = sign (d) none of these.

   (ii) If two constraints do not intersect in the positive quadrant of the graph, then
   (a) the problem is infeasible
   (b) the solution is unbounded
   (c) one of the constraints is redundant
   (d) none of these.

   (iii) The solution to a transportation problem with m-rows (supplies) and n-columns (destination) is feasible if number of positive allocations are
   (a) m + n (b) m × n (c) m + n – 1 (d) m + n + 1.

   (iv) If there were n workers and n jobs in an assignment problem , there would be
   (a) n! solutions (b) (n - 1)! solutions
   (c) (n!)n solutions (d) n solutions.

   (v) The role of artificial variables in the simplex method is
   (a) to aid in finding an initial solution.
   (b) to find optimal dual prices in the final simplex table.
   (c) to start phases of simplex method.
   (d) all of these.

(vi) Which of the following is not the characteristic of linear programming?
(a) Resources must be limited
(b) Only one objective function
(c) Parameters value remains constant during planning period
(d) The problem must be of maximization type.

(vii) If an activity has zero slack, it implies that
(a) it lies on the critical path (b) it is a dummy activity
(c) the project is progressing well (d) none of these.

(viii) Expected length of non-empty queue is given by
(a) $L = \mu / (\mu - \lambda)$ (b) $L = s\mu / (s\mu - \lambda)$
(c) $L = \lambda / (\mu - \lambda)$ (d) $L = \lambda / (\mu - \lambda) - 1/\mu$.

(ix) When total supply is equal to total demand in a transportation problem, the problem is said to be
(a) balanced (b) unbalanced
(c) degenerate (d) none of these.

(x) When no saddle point is found in a payoff matrix of a game, the value of the game is then found by
(a) knowing joint probabilities of each row and column combination to calculate expected pay-off for that combination and adding all such values
(b) reducing size of the game to apply algebraic method
(c) both of these
(d) none of these.

Group – B

2. (a) A retail store stocks two types of shirts *A* and *B*. These are packed in attractive cardboard boxes. During a week the store can sell a maximum of 400 shirts of type *A* and a maximum of 300 shirts of type *B*. The storage capacity, however, is limited to a maximum of 600 of both types combined. Type *A* shirt fetches a profit of Rs. 2/- per unit and type *B* a profit of Rs. 5/- per unit. How many of each type the store should stock per week to maximize the total profit? Formulate a mathematical model of the problem.

(b) Solve by simplex method.

$$\text{Maximize } Z = 2x_1 - 3x_2 + 5x_3$$
$$\textit{Subject to Constraints}:$$
$$x_1 + x_2 + x_3 \leq 20$$
$$2x_1 + x_2 \leq 16$$

(b) A book binder has one printing press, one binding machine and manuscripts of 7 different books. The times required for performing printing and binding operations for different books are shown below:

Book :	1	2	3	4	5	6	7
Printing time (hours) :	20	90	80	20	120	15	65
Binding time (hours) :	25	60	75	30	90	35	50

Decide the optimum sequence of processing of books in order to minimize the total time required to bring out all the books.

(2 + 2 + 2 + 2) + 4 = 12

Player A	Player B				
	B_1	B_2	B_3	B_4	B_5
A_1	2	4	3	8	4
A_2	5	6	3	7	8
A_3	6	7	9	8	7
A_4	4	2	8	4	3

6 + 6 = 12

Group – E

8. (a) Determine the optimal sequence of jobs that minimize the total elapsed time, based on the following information. Processing time on machines is given in hours and passing is not allowed.

Job :	A	B	C	D	E	F	G
Machine (M_1):	3	8	7	4	9	8	7
Machine (M_2) :	4	3	2	5	1	4	3
Machine (M_3) :	6	7	5	11	5	6	12

(b) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes. Calculate:
(i) expected queue size (line length) .
(ii) probability that the queue size exceeds 10 .

If the input of trains increases to an average of 33 per day, what will be the change in (a) and (b)?

6 + [(1.5× 2)+3] = 12

9. (a) In a departmental store one cashier is there to serve the customers. And the customers pick up their needs by themselves. The arrival rate is 9 customers for every 5 minutes and the cashier can serve 10 customers in 5 minutes. Assuming Poisson arrival rate and exponential distribution for service rate, find:
(i) Average number of customers in the system.
(ii) Average number of customers in the queue or average queue length.
(iii) Average time a customer spends in the system.
(iv) Average time a customer waits before being served.

$x_2 + x_3 \leq 10$
$x_1, x_2, x_3 \geq 0$

4 + 8 = 12

3. (a) Solve the following LP problem using Big-M method .

$Maximize\ Z = 2x_1 + x_2 + 3x_3$
$Subject\ to\ Constraints:$
$2x_1 + 3x_2 + 4x_3 = 12$
$x_1 + x_2 + 2x_3 \leq 5$
$x_1, x_2, x_3 \geq 0$

(b) Write down the dual of the following LP problem.

$Minimize\ Z = x_1 + x_2 + x_3$
$Subject\ to\ Constraints:$
$x_1 - 3x_2 + 4x_3 = 5$
$x_1 - 2x_2 \leq 3$
$2x_2 - x_3 \geq 4$
$x_1, x_2 \geq 0, x_3\ unrestricted\ in\ sign$

8 + 4 = 12

Group – C

4. (a) A company has four warehouses I, II, III and IV. It is required to deliver a product from these warehouses to three customers A, B and C. The warehouses have the following amounts in stock:

Warehouse:	I	II	III	IV
No. of units:	15	16	12	13

and the customer's requirements are

Customer:	A	B	C
No. of units:	18	20	18

The table below shows the costs of transporting one unit from warehouse to the customer.

Customer		Warehouse			
		I	II	III	IV
	A	8	9	6	3
	B	6	11	5	10
	C	3	8	7	9

i. Formulate this problem as a transportation problem in order to maximize profit.
ii. Find the initial solution using Vogel's approximation method (VAM).
iii. Test for optimality and find the optimal solution.

(b) Explain the rim condition in Transportation Problem.

(3 + 4 +3) + 2 = 12

5. (a) A company operates in four territories, and four salesmen available for an assignment. The territories are not equally rich in their sales potential. It is estimated that a typical salesman operating in each territory would bring in the following annual sales:

Territory:	I	II	III	IV
Annual sales (`) :	126000	105000	84000	63000

The four salesmen also differ in their ability. It is estimated that, working under the same conditions, their yearly sales would be proportionately as follows:

Salesmen :	A	B	C	D
Proportion :	7	5	5	4

If the criterion is maximum expected total sales, the intuitive answer is to assign the best salesman to the richest territory, the next best salesman to the second richest and so on; verify this answer by the assignment technique.

(b) Explain the difference between transportation problem and an assignment problem.

9 + 3 = 12

Group – D

6. (a) How Dummy event helps us to draw a network correctly ?What is merge event and burst event ?

(b) An established company has decided to add a new product to its line. It will buy the product from a manufacturing concern, package it, and sell it to a number of distributors that have been selected on a geographical basis. Market research has already indicated the volume expected and the size of sales force required. The steps shown in the following table are to he planned.

Activity	Description	Predecessor Activity	Duration (Days)
A	Organize sales office	—	6
B	Hire salesmen	A	4
C	Train salesmen	B	7
D	School advertising agency	A	2
E	Plan advertising campaign	D	4
F	Conduct advertising campaign	E	10
G	Design package	—	2
H	Setup packaging facilities	G	10
I	Package initial stocks	J, H	6
J	Order stock from manufacturer	—	13
K	Select distributors	A	9
L	Sell to distributors	C, K	3
M	Ship slacks 10 distributors	I, L	5

(i) Draw a network diagram for this project.
(ii) Indicate the critical path.
(iii) For each non-critical activity, find out the total and free float.

3 + (3 + 3 + 3) = 12

7. (a) Use the graphical method to obtain the optimal strategies for both persons and the value of the game for two-person zero-sum game, whose payoff matrix is as follows:

Player A	Player B	
	B1	B2
A1	1	-3
A2	3	5
A3	-1	6
A4	4	1
A5	2	2
A6	-5	0

(b) Explain the principal of dominance in game theory and solve the following game