B.TECH / CSE /3RD SEM/ CSEN 2102/2017
DISCRETE MATHEMATICS
(CSEN 2102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **$10 \times 1 = 10$**

(i) A disconnected planar graph has 6 edges, 10 vertices and 3 faces. The number of components in the graph is
(a) 6 (b) 5 (c) 3 (d) 7.

(ii) The chromatic number of a complete graph with 15 vertices is
(a) 16 (b) 15 (c) 13 (d) 17.

(iii) $p \longrightarrow (q \wedge \sim q) \equiv$
(a) $\sim p$ (b) $\sim q$ (c) F (Contradiction) (d) T (Tautology).

(iv) $\sim (\exists x)A(x) \equiv$
(a) $(\forall x) \sim A(x)$ (b) $(\exists x) \sim A(x)$
(c) $\sim(\forall x) A(x)$ (d) None of the others.

(v) Let p be the proposition 'It is cold' and q be the proposition 'It is raining'. Then the symbolic form of the statement 'It is cold or it is not raining' is
(a) $p \vee q$ (b) $\sim p \wedge q$ (c) $\sim p \vee q$ (d) $p \vee \sim q$.

(vi) The generating function for the sequence $\{1, 1, 1, 1, \ldots \ldots\}$ is
(a) $\frac{1}{1-x}$ (b) $\frac{1}{(1-x)^2}$ (c) $\frac{1}{(1-x)^3}$ (d) $\frac{1}{(1-x)^4}$.

(vii) Total number of non-negative integer valued solutions to the equation $x + y + z = 17, x, y, z \geq 0$ is
(a) 170 (b) 171 (c) 175 (d) 180 .

(viii) The chromatic number of a circuit having 37 vertices is
(a) 36 (b) 37 (c) 12 (d) 3.

(ix) How many ways can the letters of the word PICNIC be arranged?
(a) 150 (b) 180 (c) 210 (d) 250.

(x) If G is a graph and G^* is the dual of it then the number of edges in G^* is equal to
(a) the number of vertices in G (b) the number of edges in G
(c) twice the number of vertices in G (d) the number of regions in G.

Group - B

2. (a) Construct the truth table for $(r \rightarrow (p \vee \sim q)) \vee q$

(b) Without constructing a truth table, prove that
$$(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \equiv r$$
6 + 6 = 12

3. (a) State the definitions of conjunctive normal form and disjunctive normal form. Find the principal conjunctive normal form of
$$\sim ((p \wedge \sim q) \vee (p \wedge r)) \vee \sim p$$

(b) (i) Find the truth value of $'\forall x, P(x)'$ where P(x) is the statement "$x^2 < 20$" and the domain is the set {1, 2, 3, 4}.

(ii) Determine the truth value of the quantifier $'\exists x, x^2 - 2x + 5 = 0'$ where the domain is the set of all real numbers.

(2 + 4) + (3 + 3) = 12

Group - C

4. (a) Four distinguishable dice are thrown simultaneously. In how many ways a total of 14 can be obtained?

(b) Find the number of integers between 1 and 1000, inclusive, that are not divisible by any of the numbers 5, 6 and 8.

6 + 6 = 12

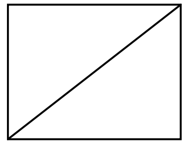
5. (a) Apply generating function technique to solve the following recurrence relation: $a_{n+2} - 4a_n = 0$ for $n \geq 0$; $a_0 = 0, a_1 = 1$.

(b) Find the general solution of the following recurrence relation:
$y_{n+2} - y_{n+1} - 2y_n = n^2$ for $n \geq 0$

6 + 6 = 12

Group – D

6. (a) State the Decomposition Theorem. Apply this theorem to find the chromatic polynomial of the following graph. Show your work in detail.



(b) Find whether the polynomial $\lambda^3 - 5\lambda^2 + 3\lambda$ is a possible chromatic polynomial of some non-null graph.

(2 + 6) + 4 = 12

7. (a) State Hall's Marriage Theorem.

(b) Define matching and perfect matching. Write down all the perfect matchings in K_4, a complete graph having four vertices. (You may name the vertices a, b, c, d.)

3 + (2+ 2 + 5) = 12

Group – E

8. (a) (i) A planar graph has degree sequence as {2, 2, 2, 3, 3, 3, 4, 4, 5}. How many faces will it have?

(ii) Can $x^4 - 4x^3 + 7x^2 - 2x + 3$ be a chromatic polynomial? Justify your answer.

(b) Applicants a_1, a_2, a_3, and a_4 apply for five posts p_1, p_2, p_3, p_4 and p_5. The applications are done as follows: $a_1 \to \{p_1, p_2\}$, $a_2 \to \{p_1, p_3, p_5\}$, $a_3 \to \{p_1, p_2, p_3, p_5\}$ and $a_4 \to \{p_3, p_4\}$. Use Hall's Marriage Theorem to determine whether every applicant can be offered a post. Show your work in detail.

(4 + 2) + 6 = 12

9. (a) Find the number of non-negative integer valued solutions of the equation $x_1 + x_2 + x_3 + x_4 = 15$ where

(i) $x_1 \geq 8$ and $x_2, x_3, x_4 \geq 0$

(ii) $0 \leq x_1 < 8$ and $x_2, x_3, x_4 \geq 0$

(b) (i) Of any 26 points within a rectangle measuring 20 cm by 15 cm, show that at least two are within 5 cm. of each other.

(ii) Determine how many strings can be formed by arranging the letters A, B, C, D, E such that neither the pattern AB nor the pattern BE appears.

(3 + 3) + (3 + 3) = 12