(vi) Which of the following functions is asymptotically smallest?

(a) 2^n (b) $n^{\log n}$ (c) $n^{\sqrt{n}}$ (d) $\sqrt[3]{\log n}$

(vii) Worst case time complexity for inserting an element in a sorted array so that it stays sorted is

(a) O(1) (b) O(n)
(c) $O(n^2)$ (d) none of these.

(viii) The 2nd largest of n elements can be found in X number of comparisons. X =

(a) 2(n – 1) (b) $n + \lceil \lg n \rceil - 2$
(c) n + lg n (d) $3/2 \lceil n \rceil$

(ix) In the algorithm for pattern matching using Finite Automata, the suffix function σ(x) is the _____est _____ of the pattern P that is also a _____ of x.

(a) large, prefix, suffix (b) small, prefix, suffix
(c) large, suffix, prefix (d) large, suffix, prefix.

(x) Topological Sort cannot be done in a directed graph of n vertices if the graph contains

(a) more than n edges (b) contains a directed cycle
(c) contains a rooted tree (d) all of the above.

Group – B

2. (a) Write a recursive algorithm to compute X^n and show that the worst case time complexity of your algorithm is O(log n).

(b) Consider the following recurrence: T(n) = T(n/3) + T(2n/3) + cn. Use recursion tree to determine a good asymptotic upper bound of the given recurrence.

(c) Do an average case analysis for binary search on a sorted array of n numbers.

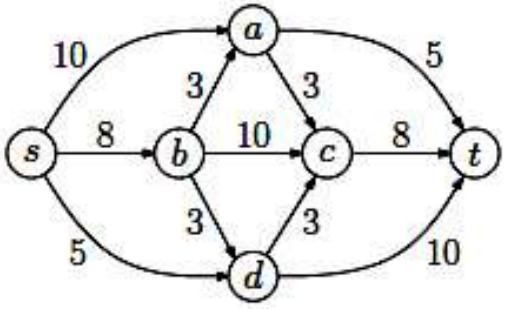
(2 + 2) + 3 + 5 = 12

3. (a) Mr. Nano is a VLSI engineer. He wants to connect **n** circuit points to the clock signal. Now, the clock signal is going to pass parallel to the x-axis and all those circuit points are going to be connected by wires which are all vertical to the clock line. Suppose the widths of the vertical wires in the given diagram are also varying. Please look at the adjoining figure to have a feel. Now, if the coordinates (x_i, y_i) for each circuit point c_i to be connected are given, find $y = y_c$ through which the clock line should pass to minimize

$$L = \sum_{i=1}^{n} w_i(y_i - y_c),$$

Group – E

8. (a) Define a flow network. Apply FORD-FULKERSON algorithm on the following flow network to find the maximum flow in the network. s & t denotes source & destination and the weights associated with every edge represents capacity of the respective edge.



(b) Prove that, using the linked-list representation of disjoint sets and the weighted-union heuristic, a sequence of m MAKE-SET, UNION and FIND-SET operations, n of which are MAKE_SET operations, takes O(m + nlgn) time.

(2 + 6) + 4 = 12

9. (a) Show that the Clique Decision Problem is NP-hard by using the result that the 3-CNF-SAT problem is computationally hard.

(b) Define the following terms in the context of optimization problems -

(i) Polynomial-time approximation scheme
(ii) Fully polynomial-time approximation scheme (FPTAS).

(c) (i) Fill in the blank: "No _____ approximation algorithm exists for a general TSP."

(ii) Give a 2-approximation algorithm for a TSP that satisfies triangle inequality and give a small proof that it indeed maintains the approximation ratio.

5 + 2 + (3 + 2) = 12

DESIGN & ANALYSIS OF ALGORITHMS
(CSEN 2201)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

   (i) Input comprises a sorted list of n integers with many duplications such that the number of distinct integers in the sequence is O (log n), the time complexity to find an element in the list is given by
   (a) O(n) (b) O(log n) (c) O(log log n) (d) θ(1).

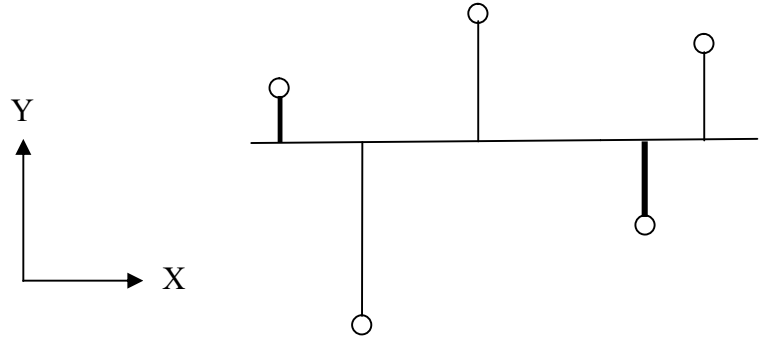
   (ii) Time complexity for the Floyd's algorithm to find all pairs of shortest path of a graph G with V vertices & E edges using dynamic programming based method is
   (a) $O(V^2)$ (b) $O(E^2)$ (c) $O(V^3)$ (d) $O(E^2 \log V)$.

   (iii) Which one of these statements is false?
   (a) All NP-Hard problems are NP-Complete
   (b) All NP-Complete problems are NP-Hard
   (c) There exists some NP-Complete problems which are not NP-Hard
   (d) Both (a) and (c).

   (iv) Which of the following problem is polynomial-time solvable?
   (a) Longest Path problem
   (b) Finding a Hamiltonian circuit in a given graph
   (c) Finding an Eulerian circuit in a given graph.
   (d) None of the above.

   (v) The time complexity of counting sort with number of elements n and maximum element m is
   (a) O(m log m) (b) O(n log m) (c) O(n + m) (d) O(n – m).

(so here you are minimizing the total material may be gold for excellent signal speed used to route these vertical wires). Also give a simple proof that the answer you found is optimal.



(b) Show that the asymptotic lower bound for comparison based sorting algorithms is $\Omega(n \log_2 n)$. You may use a small diagram for explanation.

(3 + 5) + 4 = 12

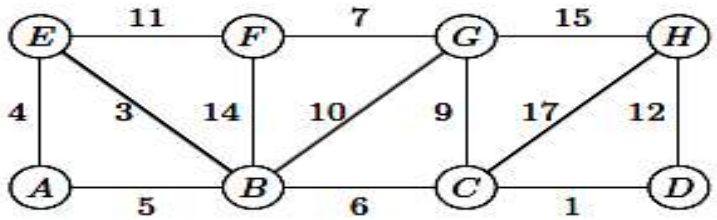
Group – C

4. (a) Matrices A, B and C, of dimensions 10 × 25, 25 × 6 and 6 × 15 respectively, are provided. You are told to ensure that when computing the matrix product A . B . C the number of scalar multiplications will be a minimum. How would you parenthesize the product and what would be the minimum number of scalar multiplications you would need to perform?

(b) Given a sequence of numbers A = {a_1, a_2, …., a_n}, design a dynamic programming based algorithm to find the length of the largest subset such that for every i < j, $a_i < a_j$. Find the complexity of your algorithm. (For example if a sequence of numbers is A = {11, 17, 5, 8, 6, 4, 7, 12, 3}, then the length of the largest subset is 4 and the set is 5, 6, 7, 12).

3 + (6 + 3) = 12

5. (a) Define a spanning tree. Consider the weighted graph below. Apply Priority Queue based Prim's algorithm on this graph starting from vertex A to compute the minimum weight spanning tree (MST). Write the edges in the order in which they are added to the MST.



(b) Find an optimal Huffman code for the following set of 6 characters using Greedy strategy. Frequencies of the characters are given as follows:

a : 05 b : 48 c : 07 d : 17 e : 10 f : 13

Also explain the strategy briefly.

(1 + 6) + (3 + 2) = 12

Group – D

6. (a) A sequence of *n* operations is performed on a data structure. The *i*th operation costs

$2i$ if *i* is an exact power of 2 but not of 4,
$4i$ if *i* is an exact power of 4,
1 otherwise.

Use aggregate analysis to determine the value of k such that the amortized cost per operation lies between k and k + 1.

(Hint: For operation i = 1 - cost is 1,
operation i = 2 - cost is 4,
operation i = 3 - cost is 1,
operation i = 4 - cost is 16,
operation i = 5, 6, 7 - cost is 1,
operation i = 8 - cost is 16, and so on)

(b) Draw a state-transition diagram for string-matching automaton for the following pattern P over alphabet Σ = {a, b}. (Be careful about careless mistakes as this is a simple question).

P = abbabaabbabba

7 + 5 = 12

7. (a) Suppose that all characters in the pattern P are different. Show how to accelerate the NAÏVE String Matching algorithm to run in time O(n) for searching the pattern P on an n-character text T.

(b) (i) Prove that subpaths of shortest paths are also shortest paths.
(ii) When does a single-source shortest path problem become undefined?
(iii) What is the asymptotic time complexity of Bellman Ford Algorithm?
(iv) Mention the name of three different data structures that you can use to implement the priority queue required in Dijkstra's algorithm. What is the time complexity in each of those 3 cases both for dense graphs and sparse graphs?

5 + (2 + 1 + 1 + 3) = 12