(viii) Solution of the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_0 = 0$ is
(a) $1 - 2^n$ (b) $2^n - 2$ (c) $2^{n-1} - 1$ (d) $2^n - 1$.

(ix) If a simple connected graph has at least one edge, then the sum of the coefficients in its chromatic polynomial is
(a) 1 (b) 2 (c) 3 (d) 0.

(x) A simple connected planar graph with the same number of vertices and edges determines
(a) 1 region (b) 3 regions
(c) 2 regions (d) none of the above.

Group - B

2. (a) Show that $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

(b) Obtain the DNF of $p \longrightarrow (p \longrightarrow q) \wedge \{\sim (\sim q \vee \sim p)\}$

6 + 6 = 12

3. (a) Find whether the argument given below is valid or not.
If Lisa's job performance for the year is good, she will get a bonus. If she gets a bonus, she will take a vacation. If she takes a vacation, she will take a cruise. Lisa did not take a cruise. Therefore Lisa did not get a bonus.

(b) Symbolize using quantifiers, predicates and logical connectives:
All birds can fly.

7 + 5 = 12

Group - C

4. (a) Use the method of generating function to solve the recurrence relation $a_n = 3a_{n-1} + 1$ for $n \geq 1$, given that $a_0 = 1$.

(b) How many solutions does the equation $x_1 + x_2 + x_3 = 13$ have, where x_1, x_2, x_3 are non-negative integers less than 6?

6 + 6 = 12

5. (a) Find the number of positive integers less than 10,00,000 the sum of whose digits is equal to 19 ? Justify your answer.

(b) 5 balls are to be placed in 3 boxes. Each box can hold all the 5 balls. In how many different ways can we place the balls so that no box is left empty if
(i) balls and boxes are different.

(ii) balls are identical and boxes are different.

6 + 6 = 12

Group – D

6. (a) Prove Euler's formula: A simple connected planar graph G with n vertices and e edges has $f = e - n + 2$ regions.

(b) State the definition of vertex connectivity of a simple graph. Find the vertex connectivity of $K_{m,n}$. Give reasons for your answer.

6 + 6 = 12

7. (a) Prove that the chromatic polynomial of a simple graph is a polynomial.

(b) If G is the connected graph obtained by linking two triangles so that they have one vertex in common, find the chromatic polynomial of G. Give reasons for your answer in detail and show your calculations.

6 + 6 = 12

Group - E

8. (a) State the definition of the chromatic number of a simple graph. Prove that every simple graph with 6 vertices whose chromatic number is 3 has at most 12 edges.

(b) Let G be a simple connected planar graph with $n \geq 3$ vertices, e edges and f regions (faces).
(i) Prove that $e \geq \frac{3}{2} f$
(ii) If the triange graph K_3 is not a subgraph of G, then prove that $e \leq 2n - 4$.

6 + (3 + 3) = 12

9. (a) Show that among $(n + 1)$ arbitrarily chosen integers, there must exist two whose difference is divisible by n.

(b) Show that $\{(p \wedge \sim q) \longrightarrow r\} \longrightarrow \{p \longrightarrow (q \vee r)\}$ is a tautology, by constructing a truth table.

6 + 6 = 12

DISCRETE MATHEMATICS
(CSEN 2102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Which one of the following is not true for the chromatic polynmial $f(G,x)$ having chromatic number m:
(a) $f(G,m) \neq 0$ (b) $f(G,m-1) \neq 0$
(c) $f(G,m+1) \neq 0$ (d) $f(G,m+2) \neq 0$.

(ii) $\sim(p \wedge q) \equiv$
(a) $\sim p \wedge \sim q$ (b) $p \vee q$
(c) $\sim p \vee \sim q$ (d) none of these.

(iii) If a planar graph determines 10 regions then the number of vertices in its dual is
(a) 8 (b) 9 (c) 10 (d) 11.

(iv) The generating function for the sequence $\{0,1,0,1,0,1, \ldots.\}$ is
(a) $\frac{x^3}{1-x}$ (b) $\frac{x}{1-x^2}$ (c) $\frac{1}{1-x^2}$ (d) $\frac{1}{1-x}$.

(v) If the truth value of the propositions p, q and r are T, T and F respectively then the truth value of $p \wedge (q \vee r)$ is
(a) F (b) T
(c) both F and T (d) none of these.

(vi) How many ways can the letters of the word PICNIC be arranged?
(a) 720 (b) 120 (c) 180 (d) none of these.

(vii) The chromatic number of a graph having 6 vertices and no edges is
(a) 6 (b) 5 (c) 0 (d) 1.