PHYSICS – I
(PHY1001)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) Which of the following represent a central force?

(a) $\vec{F} = kcos^2\theta\ \hat{r}$ (b) $\vec{F} = kr^2\ \hat{r}$

(c) $\vec{F} = k\frac{\cos^2\theta}{r^2}\hat{r}$ (d) $\vec{F} = k\frac{1}{r^2}\hat{\theta}$

(ii) Which of the following is a direct consequence of the fact that the gravitational force is a central force?

(a) The orbit of every planet must be a perfect circle.
(b) The linear momentum of a planet is conserved.
(c) The angular momentum of a planet is conserved.
(d) The speed of the planet in its orbit is constant.

(iii) The resonant frequency of an electrical LCR circuit is given by

(a) $\nu = 2\pi\sqrt{LC}$ (b) $\nu = \frac{1}{2\pi\sqrt{LC}}$

(c) $\nu = \frac{2\pi}{\sqrt{LC}}$ (d) $\nu = \frac{1}{2\pi}\sqrt{\frac{L}{C}}$

(iv) A function $x(t) = Asin^2\omega_o t$ represents

(a) SHM about a point $x = \frac{A}{2}$ with time period $\frac{\pi}{\omega_0}$
(b) SHM about a point $x = \frac{A}{2}$ with time period $\frac{2\pi}{\omega_0}$
(c) SHM about a point $x = 0$ with time period $\frac{\pi}{\omega_0}$
(d) SHM about a point $x = 0$ with time period $\frac{2\pi}{\omega_0}$

(v) The displacement of a moving particle at any time t is given by $y = 5sin3\pi t + 5\sqrt{3}cos3\pi t$. What is the amplitude of the motion?

(a) 5 (b) 10 (c) 100 (d) $5\sqrt{3}$

(vi) A half wave plate introduces a phase difference
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{5}$ (c) 2π (d) π

(vii) A plane electromagnetic wave propagates along the $+x$ direction in free space. Which of the following represents a possible set of components for the electric field vector $\vec{E}$ and the magnetic field vector $\vec{B}$?
(a) E_z, B_z (b) E_x, B_y (c) E_y, B_z (d) E_y, B_x

(viii) Which of the following commutation relation may result to the uncertainty in measurement?
(a)$[X, P_x] = i$ (b)$[Y, P_z] = 0$ (c) $[P_y, P_z] = 0$ (d) $[Y, Z] = 0$

(ix) For a particle in a box $0 \leq x \leq a$ the expectation value of momentum is
(a) 1 (b) ½ (c) 1/3 (d) 0

(x) For a particle of mass m in a one-dimensional infinite square well of width L (from $x = 0$ to $x = L$ what is the energy of the first excited state ($n = 2$)?
(a) $\frac{2h^2}{mL^2}$ (b) $\frac{2h^2}{8mL^2}$ (c) $\frac{h^2}{2mL^2}$ (d) $\frac{4h^2}{mL^2}$

Fill in the blanks with the correct word

(xi) For a series LCR circuit with AC voltage in resonance the current is _____ phase with the voltage.

(xii) The unit of damping factor is_____.

(xiii) The light vector of a polarized light is given by $\vec{E}(y,t) = E_0 \cos(ky - \omega t)\,\hat{k}$ is _____ polarized light.

(xiv) If the orbit equation is given by $r = e^{b\theta}$, the force law is given by _____.

(xv) If two operators $\widehat{A}$ and $\widehat{B}$ commute, they possess a complete set of simultaneous _____.

Group - B

2. (a) Starting from the differential equation of orbit, justify that in absence of any external force a planet will move in a straight-line orbit. *[(CO1)(Evaluate/HOCQ)]*

(b) Obtain the expression for acceleration in plane polar co-ordinate system? *[(CO4)(Remember/LOCQ)]*

(c) The equation of the of orbit for a planet moving under the influence of central force is given by $r = 3\,e^{5\theta}$, find the magnitude of the areal velocity. *[(CO5)(Apply/IOCQ)]*

(d) The orbit of a particle under the influence of a central force is given by $r = e^{-\theta}$. Find out the corresponding force law. *[(CO5)(Remember/LOCQ)]*

3 + 3 + 3 + 3 = 12

3. (a) Prove that the speed v of a particle moving in an elliptical path in an inverse square force is given by $v^2 = \frac{K}{m}\left(\frac{2}{r} - \frac{1}{a}\right)$ where a is the semi-major axis. *[(CO3)(Analyse/HOCQ)]*

(b) A particle of mass m moves in a central potential $V(r) = \frac{-k}{r}$ in an elliptical orbit $r(\theta) = \frac{a(1-\epsilon^2)}{1+\epsilon\cos\theta}$ (symbols have usual meaning). If the total energy of the particle is $E = \frac{-k}{2a}$, find the maximum kinetic energy of the particle. *[(CO4)(Remember/LOCQ)]*

(c) Is the $\vec{F} = \cos\theta\,\hat{r}$ a central force? Explain using the help of definition of central force. *[(CO1)(Apply/IOCQ)]*

5 + 5 + 2 = 12

Group - C

4. (a) Starting from the consideration that the rate of loss of total energy of a damped harmonic oscillator is equal to the power dissipated by the damping force, find out the differential equation of motion of the damped pendulum. *[(CO3)(Analyse/HOCQ)]*

(b) The displacement of a particle of mass 3 gm undergoing damped oscillation is given by
$x(t) = 10e^{-0.5t} sin\ 3t$ cm.
(i) Find the velocity of the particle at t = π/6 sec
(ii) Find the logarithmic decrement and relaxation time of the system.
[(CO4)(Remember/LOCQ)]

(c) The damped motion of a particle is given by the following equation

$$4\frac{d^2x}{dt^2} + 6\pi\frac{dx}{dt} + \beta\pi^2 x = 0, \beta > 0$$

(i) Find the conditions for large, critical and weak damping in relation to the values of β.
(ii) If the motion starts from a displacement x(0)=-5 cm with a velocity v(0)=3 cm/sec, comment on whether the particle will ever pass through the origin in a critically damped case. *[(CO2)(Apply/IOCQ)]*

3 + (2 + 2) + (3 + 2) = 12

5. (a) Write down the differential equation of a series LCR circuit driven by a sinusoidal voltage $V(t) = V_0\cos\omega t$.
(i) Compare the differential equation with a mechanical system undergoing forced vibration with the force $f(t) = f_0\cos\omega t$.
(ii) Find out the condition for which the current and the voltage are $\frac{\pi}{2}$ out of phase. *[(CO1)(Analyse/HOCQ)]*

(b) Show that power resonance frequency is independent of resistance. *[(CO6)(Remember/LOCQ)]*

(c) Draw the velocity resonance curve for three different values of damping factor. *[(CO1)(Apply/IOCQ)]*

(d) Describe the meaning of impedance in case of forced oscillation in series LCR circuit. *[(CO1)(Understand/LOCQ)]*

(2 + 2) + 3 + 3 + 2 = 12

Group - D

6. (a) Obtain the condition for constructive and destructive interference for superposition of two light wave vectors. *[(CO3)(Analyse/HOCQ)]*

(b) Obtain the expression for the diameter of n^{th} dark ring in Newton's ring experiment. *[(CO4)(Remember/LOCQ)]*

(c) How can one convert an elliptically polarized light into a plane polarized light and vice-versa. *[(CO2)(Apply/IOCQ)]*

4 + 4 + 4 = 12

7. (a) A right circularly polarized beam of light (λ = 5250Å) is incident normally on adoubly refracting crystal with optic axis parallel to its surface. The thickness of thecrystal is 0.00075 mm. The difference between the refractive indices due to the E-wave and O-wave is 0.175. Find the state of polarization of the emergent light beam. *[(CO3)(Remember/LOCQ)]*

(b) Differentiate between circularly polarized light and un-polarized light? How can you identify whether a light is circularly polarized or un-polarized light? *[(CO4)(Remember/LOCQ)]*

(c) What is a half-wave plate? A linearly polarized wave passes through a half-wave plate made of positive crystal with $n_0 = 1.5$. The minimum thickness of the plate is twice the wave length used. Obtain the value of n_e. *[(CO2)(Remember/LOCQ)]*

4 + (2 + 2) + (2 + 2) = 12

Group - E

8. The quantum state of a particle confined within $0 \leq x \leq \infty$ is given by $\psi(x) = A\sqrt{x}e^{-2x+i\omega t}$.

(i) Evaluate the normalization constant and the probability density.

(ii) Find the expectation value of $\frac{1}{x}$.

(iii) Find the point where the probability density is maximum. *[(CO5)(Evaluate/HOCQ)]*

[(3 + 3) + 3 + 3] = 12

9. (a) Define self-adjoint (Hermitian) operator with an example. *[(CO2)(Understand/LOCQ)]*

(b) Show that eigenvalue of a self-adjoint operator is real and eigenstates corresponding to two different eigenvalues are orthogonal. *[(CO1)(Remember/LOCQ)]*

4 + (4 + 4) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	50	17.7	32.3