

MATHEMATICS II
(MTH1201)

Time Allotted : 2½ hrs

Full Marks : 60

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
any 4 (four) from Group B to E, taking one from each group.*

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve:

12 × 1 = 12

Choose the correct alternative for the following

- (i) If X and Y are independent random variables then
(a) $E(XY) = E(X) + E(Y)$ (b) $E(XY) = E(X) - E(Y)$
(c) $E(XY) = E(X)E(Y)$ (d) $E(XY) = E(X)/E(Y)$.
- (ii) If for a random variable X , $Var(X) = 1$, then $Var(-3X + 4)$ is
(a) 4 (b) 13 (c) 9 (d) -3.
- (iii) A random variable X has the probability density function $f(x)$ as given below:
$$f(x) = \begin{cases} a + bx & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

If the expected value $E(X) = \frac{2}{3}$ then $P(X < 0.5)$ is
(a) 0.5 (b) 1 (c) 0.25 (d) 0.
- (iv) Let $f(x) = 0$ be an equation such that $f(a)f(b) > 0$ for two real numbers a and b then which one of the following is true:
(a) at least one root of $f(x) = 0$ lies in (a, b)
(b) no root of $f(x) = 0$ lies in (a, b)
(c) an odd number of roots of $f(x) = 0$ lies in (a, b)
(d) either no root or an even number of roots of $f(x) = 0$ lies in (a, b) .
- (v) Gauss-elimination method fails when any one of the pivotal elements is
(a) 0 (b) 1 (c) 2 (d) -1.
- (vi) System of linear equations may be solved by
(a) Newton-Raphson method (b) Gauss-Seidel method
(c) Euler's method (d) Runge-Kutta method.
- (vii) The maximum degree of any vertex in a simple graph with 10 vertices is
(a) 10 (b) 5 (c) 20 (d) 9.

- (viii) The number of odd vertices in a graph is always
 (a) odd (b) even (c) multiple of 3 (d) multiple of 5.
- (ix) If $\mathcal{L}\{F(t)\} = \frac{1}{s+1}$ then $\mathcal{L}\{f(3t)\}$ is
 (a) $\frac{3}{s+3}$ (b) $\frac{1}{s+3}$ (c) $\frac{1}{3s+1}$ (d) $\frac{3}{3s+1}$.
- (x) The value of $\Gamma\left(\frac{3}{2}\right)$ is
 (a) $\sqrt{\pi}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\sqrt{\pi}}{2}$

Fill in the blanks with the correct word

- (xi) Let X be a random variable which follows normal distribution with mean zero and variance one. Then $E(|X|)$ is _____.
- (xii) Laplace transform of $f(t) = te^{2t}$ is _____.
- (xiii) The value of $\int_0^{\infty} e^{-x} x^{\frac{3}{2}} dx$ is _____.
- (xiv) The chance that a leap year selected at random will contain 53 Wednesdays is _____.
- (xv) A man draw at random 3 balls from a bag containing 6 blue and 5 green balls. The probability of getting the balls all blue is _____.

Group - B

2. (a) Two containers contain 2 red, 5 black, 7 green and 1 red, 9 green, 4 black balls respectively. One ball is drawn from each container. Find the probability that both are of same colour.
 [(MTH1201.1, MTH1201.2)(Understand/LOCQ)]
- (b) The distance (in kilometres) travelled by a cyclist in a day is a continuous random variable X whose cumulative distribution function (c.d.f.) is given by:

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^3}{216}, & 0 \leq x < 6 \\ 1, & x \geq 6. \end{cases}$$

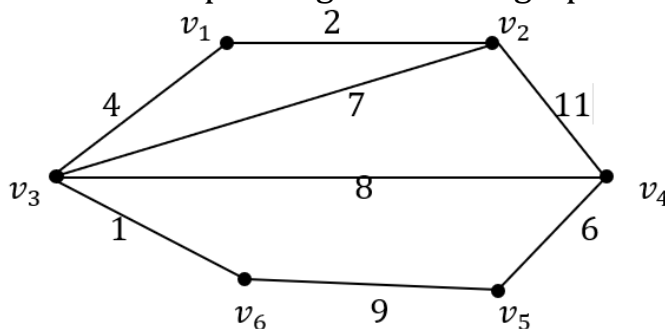
 What is the average distance travelled by the cyclist? Further find the standard deviation of the distance travelled by the cyclist from the average distance.
 [(MATH1201.1, MATH1201.2)(Analyse/IOCQ)]
6 + 6 = 12
3. (a) The probability density function of a random variable X is $f(x) = k(x-1)(2-x)$ for $1 \leq x \leq 2$. Determine
 (i) the value of k (ii) $P\left(\frac{5}{4} \leq X \leq \frac{3}{2}\right)$.
 [(MTH1201.1, MTH1201.2)(Understand/LOCQ)]
- (b) An insurance company insured 2000 scooter drivers, 5000 car drivers, 6000 truck drivers and the probabilities of accidents are respectively 0.01, 0.04 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?
 [(MTH1201.1, MTH1201.2)(Understand/LOCQ)]
6 + 6 = 12

Group - C

4. (a) Compute the values of unknown in the system of equations by Gauss-Jordan's matrix inversion method.
 $x + 3y + 2z = 17$
 $x + 2y + 3z = 16$
 $2x - y + 4z = 13.$ [[MTH1201.3](Evaluate/HOCQ)]]
- (b) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition $y(0) = 1$. Evaluate $y(0.1)$ by Euler's method, correct up to 4 decimal places, taking step length $h = 0.02$.
[[MTH1201.3](Evaluate/HOCQ)]]
6 + 6 = 12
5. (a) Compute one root of $e^x - 3x = 0$ correct to two decimal places which lies between 1 and 2 using bisection method. [[MTH1201.3](Understand/LOCQ)]]
- (b) Solve the following system of equations using Gauss – Seidel method up to three significant figures.
 $x + y + 4z = 9$
 $8x - 3y + 2z = 20$
 $4x + 11y - z = 33.$ [[MTH1201.3](Apply/IOCQ)]]
6 + 6 = 12

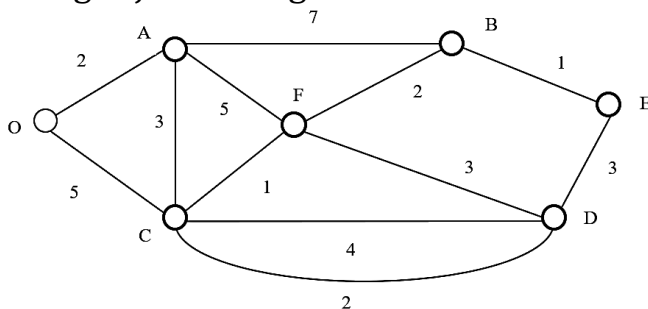
Group - D

6. (a) Prove that the number of odd degree vertices in an undirected graph is always even. [[MTH1201.4](Understand/LOCQ)]]
- (b) Is it possible to construct a graph with 15 vertices out of which 5 vertices have degree 2 and the other vertices have degree 3? [[MTH1201.4](Understand/LOCQ)]]
- (c) Find the shortest spanning tree in the graph below by applying Prim's algorithm.



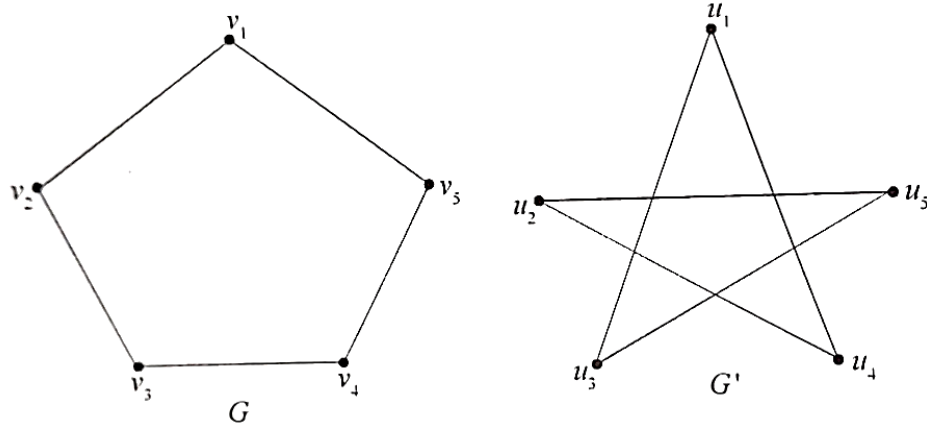
[[MTH1201.4](Apply/IOCQ)]]
3 + 3 + 6 = 12

7. (a) Using Dijkstra's algorithm find a shortest path from graph from O to E:



[[MTH1201.4](Apply/IOCQ)]]

- (b) Examine, whether the following two graphs are isomorphic :



[(MTH1201.4)(Understand/LOCQ)]

7 + 5 = 12

Group - E

8. (a) Using Gamma function show that $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} d\theta = \frac{\pi}{\sqrt{2}}$.
[(MTH1201.5, MTH1201.6)(Understand/LOCQ)]

- (b) Express the following function in the form of unit step function and find its Laplace Transform.

$$f(t) = \begin{cases} 0, & 0 < t < 1 \\ t - 1, & 1 < t < 2. \\ 1, & t > 2 \end{cases}$$

[(MTH1201.5, MTH1201.6)(Remember/LOCQ)]

6 + 6 = 12

9. (a) Evaluate $\mathcal{L}^{-1} \left\{ \frac{1}{s^2(s+1)^2} \right\}$ by using the convolution theorem.
[(MTH1201.5, MTH1201.6)(Evaluate/HOCQ)]

- (b) Solve the differential equation using the method of Laplace transform:

$$y''(t) + 9y(t) = \cos 2t \text{ where } y(0) = 1, y\left(\frac{\pi}{2}\right) = -1.$$

[(MTH1201.5, MTH1201.6)(Evaluate/HOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	48.96	26.04	25