COMPUTATIONAL MATHEMATICS
(MTH3221)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) $\Delta\left(x^{\underline{3}}\right) =$

(a) $2x^2 - 3x$ (b) $3x^2 + 2x$

(c) $3x^2 - 3x$ (d) $3x^2 + 3x$

(ii) $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \cdots =$

(a) $\frac{511}{256}$ (b) 3 (c) $\frac{15}{8}$ (d) 2

(iii) $S_2(4,2) =$

(a) 6 (b) 8 (c) 7 (d) 9

(iv) The Eulerian number $E(4,2)$

(a) 11 (b) 26 (c) 4 (d) 302

(v) $30!\mod 31 =$

(a) 31 (b) 1 (c) 29 (d) 30

(vi) $\left|\left\lceil\sqrt[4]{3}\right\rceil + \sqrt[4]{3}\right| =$

(a) 1 (b) 2 (c) 3 (d) 4

(vii) The greatest common divisor of the Fibonacci numbers F_{15} and F_{16} is

(a) 1 (b) 3 (c) 2 (d) 4

(viii) The generating function of the sequence $\{1, 2, 4, 8, 16, \ldots, 2^n, \ldots\}$ is

(a) $\frac{1}{1-x^3}$ (b) $\frac{1}{1+x^3}$ (c) $\frac{1}{1+2x}$ (d) $\frac{1}{1-2x}$

(ix) The generating function of the sequence $1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, \ldots$ is

(a) $\frac{1}{(1-x)^2}$ (b) $\frac{1}{1-x^2}$ (c) $\frac{1}{1+x^2}$ (d) $\frac{1}{(1+x)^2}$

(x) $\sum_{0\le n^2\le 5} \frac{1}{2n^2+1} =$

(a) $\frac{6508}{3465}$ (b) $\frac{17}{9}$ (c) $\frac{13}{9}$ (d) $\frac{563}{315}$

Fill in the blanks with the correct word

(xi) The number of moves necessary in the Tower of Hanoi problem for 16 discs is _____.

(xii) The value of $\lfloor \pi - 9 \rfloor$ is _____.

(xiii) The seventeenth Bernoulli number B_{17} is _____.

(xiv) The generating function of the infinite sequence $1, 1, 1, \ldots$ is _____.

(xv) $x^{\overline{4}}$ is the polynomial _____.

Group - B

2. (a) Use the perturbation technique to prove that $\sum_{0\le k\le n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

[(MATH3221.1, MATH3221.5, MATH3221.6) (Analyse/IOCQ)]

(b) Evaluate the sum $\sum_{k=1}^{n} \frac{(-1)^k k}{4k^2-1}$ (Your answer should be a function of n.)

[(MATH3221.1, MATH3221.5, MATH3221.6) (Remember/LOCQ)]

6 + 6 = 12

3. (a) Recall that $\sum_{0\le k<n} k^{\underline{m}} = \frac{k^{\underline{m+1}}}{m+1} = \frac{n^{\underline{m+1}}}{m+1}$, for integers $m, n \ge 0$.

Use this result to prove that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$.

[(MATH3221.1, MATH3221.5, MATH3221.6) (Analyze/IOCQ)]

(b) Solve the following recurrence

$P_0 = \beta - 1, P_1 = \beta \ne 1, P_n = \frac{1+P_{n-1}}{P_{n-2}}$. Find P_{100} and P_{102}.

[(MATH3221.1, MATH3221.5, MATH3221.6) (Evaluate/HOCQ)]

6 + 6 = 12

Group - C

4. (a) Let B_n denote the n-th Bernoulli number. Prove that

$B_3 = B_5 = B_7 = B_9 = B_{11} = B_{13} = B_{15} = \cdots = B_{2r+1} = \cdots = 0, r \ge 1$

[(MATH3221.2, MATH3221.5, MATH3221.6) (Analyse/IOCQ)]

(b) Compute $S_2(4, 2), S_2(4, 3), S_2(5, 2), S_2(5, 3)$. State any result that you use and show your calculations. *[(MATH3221.2, MATH3221.5, MATH3221.6) (Apply/IOCQ)]*

6 + 6 = 12

5. (a) Let $C(n, k)$ denote the number of ways of choosing k objects from n objects. Prove that $C(n, 0) + C(n, 1) + C(n, 2) + C(n, 3) + \ldots\ldots + C(n, n) = 2^n$. State any theorem that you use. *[(MATH3221.2, MATH3221.5, MATH3221.6) (Evaluate/HOCQ)]*

(b) Let $S_2(n,k)$ denote a Stirling number of the second kind (i.e. the number of ways of partitioning the set $\{1,2,3,4,5, \dots \dots, n\}$ into k non-empty subsets). Find $S_2(4,3)$. Show your work. *[(MATH3221.2, MATH3221.5, MATH3221.6) (Apply/IOCQ)]*

6 + 6 = 12

Group - D

6. (a) Let $a \equiv b(mod\ m), c \equiv d\ (mod\ m)$. Prove that $ac \equiv bd\ (mod\ m),$ and $a^8 \equiv b^8(mod\ m)$.

[(MATH3221.3, MATH3221.5, MATH3221.6) (Apply/IOCQ)]

(b) Evaluate $\left\lfloor \lfloor m\ \alpha \rfloor \frac{n}{\alpha} \right\rfloor$ when m and n are positive integers and α is an irrational number greater than n. Show your calculations and justify your answers.

[(MATH3221.3, MATH3221.5, MATH3221.6) (Analyse/IOCQ)]

6 + 6 = 12

7. (a) Let $\varphi(n)$ denote the Euler phi function. Compute $\varphi(36), \varphi(25), \varphi(29)$. Show your work and state any result that you use.

[(MATH3221.3, MATH3221.5, MATH3221.6) (Evaluate/HOCQ)]

(b) Find the remainder in the division of $1^{101} + 2^{101} + 3^{101} + 4^{101} + 5^{101} + 6^{101}$ by 11. State any theorem that you use and show your calculations in detail.

[(MATH3221.3, MATH3221.5, MATH3221.6) (Evaluate/HOCQ)]

6 + 6 = 12

Group - E

8. (a) Find the generating function of the sequence $\{a_n\}$ where $a_n = na_{n-1}$.

[(MATH3221.4, MATH3221.5, MATH3221.6) (Apply/IOCQ)]

(b) Find the coefficient of X^{23} in $(1 + X^5 + X^9)^{10}$.

[(MATH3221.4, MATH3221.5, MATH3221.6) (Analyse/IOCQ)]

6 + 6 = 12

9. (a) Solve the recurrence relation $a_{n+2} - 6a_{n-1} + 9a_n = 3.2^n + 7.3^n, n \geq 0$.

[(MATH3221.4, MATH3221.5, MATH3221.6) (Apply/IOCQ)]

(b) Find a recurrence relation for the number of ways to make a pile of n chips using garnet, gold, red, white , and blue chips such that no two gold chips are together.

[(MATH3221.4, MATH3221.5, MATH3221.6) (Create/HOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	6.25	62.50	31.25