ADVANCED ALGORITHMS
(CSEN 5201)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The function f(n) = $10n^2$ + 10^3 is
(a) $\Theta(n^2)$ (b) $\Omega(n^2)$
(c) $O(n^2)$ (d) all of the above

(ii) How many comparisons would be required to merge two sorted lists of size m and n into a sorted list of size (m + n)?
(a) $\Theta(m + n)$ (b) $\Theta(mn)$
(c) $\Theta(n^2)$ (d) $\Theta(m^2)$

(iii) Which of the following statements is not true?
(a) We can apply binary search on a single linked list if the items in the list are sorted in ascending order.
(b) The expected number of comparisons to find an element from an array containing n elements using linear search, if the element occurs k times in the array is n/k.
(c) The average case asymptotic complexity of binary search is O(1).
(d) All of the above.

(iv) If you apply a DFS to classify the edges on an undirected graph, there will be no
(a) Back edge (b) Forward edge
(c) Cross edge (d) Neither Forward edge nor cross edge

(v) Let G be an undirected connected graph with distinct edge weights, let e_{max} be the edge with maximum weight and e_{min} be the edge with minimum weight. Which of the following statement(s) is/are FALSE?
(a) Every minimum spanning tree of G contains e_{min} (b) No minimum spanning tree contains e_{max}
(c) G has a unique minimum-spanning tree (d) None of the above

(vi) Topological sort can only run on
(a) an undirected graph (b) an undirected tree
(c) a complete directed graph (d) an acylic directed graph

(vii) An undirected graph G with n vertices is represented by an adjacency matrix where all diagonal elements are 0 and non-diagonal elements are 1. Which of the following is TRUE?
(a) G does not have a minimum spanning tree (MST). (b) G has a unique MST of cost (n - 1).
(c) G has multiple MSTs of cost (n - 1). (d) None of the above.

(viii) The necessary features to apply Dynamic Programming to solve a problem are –
(a) Dependent sub-problems (b) Independent sub-problems
(c) Optimal substructure (d) Both (a) and (c)

(ix) What is the amortized cost per operation for a sequence of n PUSH, POP, and MULTIPOP operations on an initially empty stack?
(a) O(n) (b) O(n^2)
(c) O(1) (d) None of these

(x) Which of the following is NP-complete?
(a) Travelling salesman problem, longest simple path problem
(b) Matrix-chain multiplication problem, Fraction knapsack problem
(c) Edge-cover problem, 2-CNF
(d) Finding all pairs shortest path of a graph with negative edges.

Fill in the blanks with the correct word

(xi) The Minimum Spanning Tree algorithms can be implemented on _____, connected, and weighted graphs.

(xii) If you want to prove that a problem P is NP-hard, then you need to show that, P can be reduced to any known NP-hard problem in polynomial time. The statement is _____ (True/ False)

(xiii) The minimum height of an almost complete binary tree which represents a max-heap of 1023 elements will be _____.

(xiv) Dijkstra's algorithm for finding the shortest path does not work if the graph has _____ edge costs.

(xv) The GCD(1, 0) will return _____, where GCD = Greatest Common Divisor.

Group - B

2. (a) Consider the following pseudo-code for Insertion Sort and

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ALGO- INSERTION-SORT(A)
INPUT- Array A[1..n]
1.    for j ← 2 to length[A]
2.    do    key ← A[j]  //Insert into sorted A[a….j-1]
3.          i ← j – 1
4.          while (i > 0) and (A[i] > key)
5.                do A[i+1] ← A[i]
6.                i ← i – 1
7.          A[i+1] ← key
```

(i) Assuming that line i takes c_i time to execute and the line 4 executes t_j times, when the loop-variable of the outer for loop takes the value j, can you fill up the following table for all 7 lines of pseudo-code?

Cost of ith opertation	No. of times line i gets executed
C_1	n
C_2	…..
So on	…..

(ii) What will be the value of t_j for the best-case of insertion sort?

(iii) What will be the value of t_j for the worst-case of insertion sort?

(iv) What will be the value of t_j for the average-case of insertion sort? Show the calculation.

(v) What will be the value of t_j in line 4 for each j from 2 to n, when the given input array is 2, 1, 4, 3, 6, 5, ..., n, n – 1? Give a brief justification for your answer. *[(CO2)(Understand/LOCQ)]*

(b) Let $f1(n) = 10^n$, $f2(n) = n^{1000}$, $f3(n) = n^{\lg 7}$, $f4(n) = \sqrt{n}$, $f5(n) = 7^{\lg n}$. Write the correct ordering of these functions arranged in order of increasing asymptotic complexity. Do you think there can be multiple possible answers, if yes, then write that answer too. (lg stands for log base 2) *[(CO3)(Apply/LOCQ)]*

(2 + 1 + 1 + 2 + 3) + 3 = 12

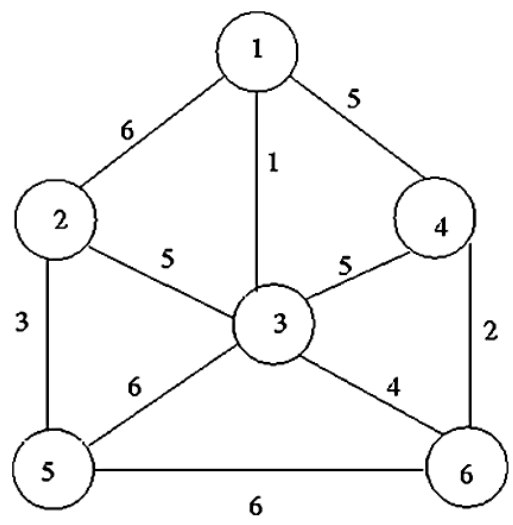
3. (a) Prove that the average-case time complexity of Quick Sort is O(nlog n) using the recurrence relation approach. *[(CO3,CO4)(Apply, Analyse/IOCQ)]*

(b) Give the pseudo-code to insert a new element in a heap. *[(CO1,CO2)(Remember, Understand/IOCQ, LOCQ)]*

7 + 5 = 12

Group - C

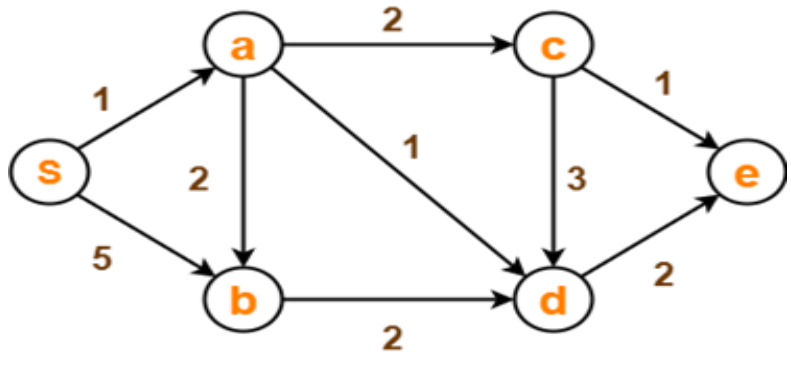
4. (a) Find out the Minimum Spanning Tree of the following graph using Prim's Algorithm with source vertex '2'. *[(CO3)(Apply/IOCQ)]*



(b) Give the pseudo-code for doing Topological Sort.
Draw a directed graph with 5 vertices and 10 edges on which you can do a Topological Sort. Label the vertices as A, B, C, D, and E. Then perform a Topological Sort on it
Can you do a Topological Sort on an undirected graph also? Justify. *[(CO2, CO3)(Apply/IOCQ)]*

5 + (2 + 1 + 3 + 1) = 12

5. (a) Run Dijkstra's algorithm on the following graph G = (V, E) to find the shortest paths from source vertex 'S'. Show each step in detail. *[(CO3)(Apply/IOCQ)]*



(b) Is the shortest path between two vertices always unique? Justify. *[(CO4)(Analyze/HOCQ)]*

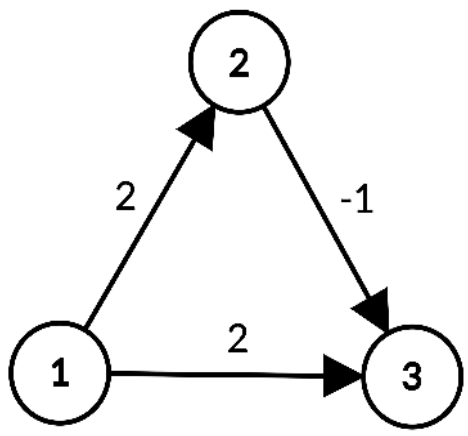
(c) Give the pseudo-code for Bellman-Ford algorithm. In finding shortest-paths in a graph, when would you prefer to use Bellman-Ford algorithm and when would you prefer to use Dijkstra's algorithm, and why? *[(CO2)(Remember,Understand/IOCQ)]*

5 + 1 + (3 + 3) = 12

Group - D

6. (a) Matrices A, B, and C, of dimensions (10 × 24), (24 × 6), and (6 × 17) respectively, are provided. You are told to ensure that when computing the matrix product A.B.C. the number of scalar multiplications will be a minimum. How would you parenthesize the product, and what would be the minimum number of scalar multiplications you would need to perform? *[(CO3)(Apply/IOCQ)]*

(b) Show each step of the execution of the Floyd-Warshall algorithm on the following graph. *[(CO3)(Apply/IOCQ)]*



(c) Give the pseudo-code for Euclid's GCD algorithm. State the complexity of the algorithm. *[(CO2)(Understand/LOCQ)]*

4 + 4 + (3 + 1) = 12

7. (a) Consider the dynamic programming algorithm for the 0/1 Knapsack problem. Consider two variants of the problem. In the first one, for n objects, the profits p(i), weights w(i), and the knapsack capacity W are all part of the input. Consider a second variant in which profits and weights are all part of the input but the capacity W is a fixed constant 100. Explain with reasons, whether the O(nW) dynamic programming algorithm represents a true polynomial time algorithm for the problem for each of the two cases. *[(CO2,CO3)(Apply/IOCQ)]*

(b) Give the outline of an O(n^2) dynamic programming algorithm that computes T(n) defined as follows

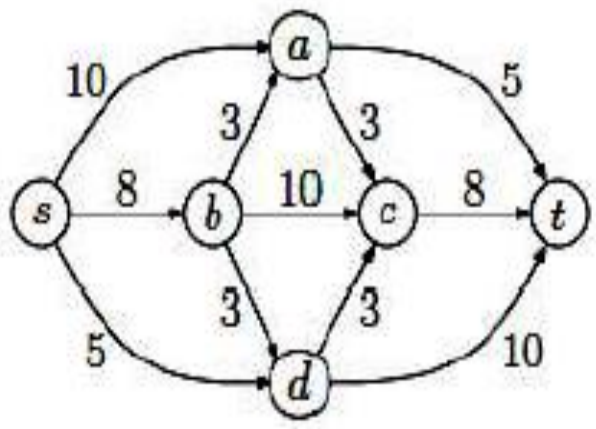
$$T(n) = 2 \sum_{i=1}^{n-1} T(i)*T(i-1)$$
$$T(0) = T(1) = 2$$

[(CO2,CO3)(Apply/HOCQ)]

(2 + 2) + (6 + 2) = 12

Group - E

8. (a) Apply the FORD-FULKERSON algorithm on the following flow network to find the maximum flow in the network. s & t denotes source & destination and the weights associated with every edge represent the capacity of the respective edge.



[(CO2,CO3)(Apply/IOCQ)]

(b) Write a 2-approximation algorithm for the Travelling Salesman Problem that will work on a graph satisfying Triangle Inequality. *[(CO2,CO3)(Apply/IOCQ)]*

7 + 5 = 12

9. (a) Fill in the four blanks with respect to the concept of NP-hardness.

A language $L \subseteq \{0, 1\}^*$ is _____ if

- ❑ $L \in$ _____ , and
- ❑ _____ $\leq_p$ _____ for every $L' \in NP$

[(CO5)(Remember/LOCQ)]

(b) Let S be an NP-complete problem and Q and R be two other problems not known to be in NP. Q is polynomial time reducible to S and S is polynomial-time reducible to R.
In the above context, for each of the two cases choose the best option possible and justify it briefly.
(i) Q is in P / Q is NP-hard / Q is NP-complete / Not enough evidence to conclude
(ii) R is in P / R is NP-hard / R is NP-Complete / Not enough evidence to conclude. *[(CO3, CO5)(Apply/IOCQ)]*

(c) Assume A and B are specific decision problems and that f is a polynomial-time function for reducing A to B. Which statement is true and which is false?

1. if A $\rightarrow_f$ B and A $\in$ P, then B $\in$ P
2. if A $\rightarrow_f$ B and B $\in$ NP, then A $\in$ P
3. if A $\rightarrow_f$ B and A $\in$ NP-Complete, then B $\in$ NP-Hard
4. if A $\rightarrow_f$ B and B $\in$ NP-Complete, then A $\in$ NP-Complete
5. if A $\rightarrow_f$ B and B $\rightarrow_f$ A, then A, B $\in$ NP-Complete
6. if A $\rightarrow_f$ B and B $\in$ NP-Complete, then B $\rightarrow_f$ A
7. if A $\rightarrow_f$ B and A $\in$ NP-Complete, then B $\in$ NP-Complete

[(CO3,CO5)(Understand/LOCQ)]

2 + (2 × 1.5) + (7 × 1) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	46.88	43.75	9.37