FINITE ELEMENT METHOD
(MECH 3231)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) In the domain residual method, what is the 'residual'?
(a) The exact solution.
(b) The difference between the approximate and exact solutions.
(c) The initial condition.
(d) The boundary condition.

(ii) Rayleigh-Ritz method is based on
(a) Principle of conservation of external forces and reaction forces.
(b) Principle of conservation of external moments and reaction moments.
(c) Principle of Stationary Total Potential.
(d) All of the above principles.

(iii) The elemental stiffness matrix of a 2-noded BAR element having cross-sectional area 'A', elemental length 'L' and modulus of elasticity 'E' is
(a) $\frac{L}{AE}\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}$ (b) $\frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$
(c) $\frac{AL}{E}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ (d) $\frac{AE}{L}\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$

(iv) For a 2-Node BEAM element which of the following statement is not true.
(a) Nodal DOF is 2 (b) Elemental DOF is 4
(c) It has four shape functions (d) Stiffness matrix is 2 × 2 matrix

(v) The elemental stiffness matrix of a BEAM element can be expressed in terms of [B] matrix, Young's modulus E and differential volume dv as
(a) $\int [B]^T\,[B]E\,dv$ (b) $\int [B]\,[B]^T E\,dv$ (c) $\int [B]E\,dv$ (d) $\int [B]\,dv$

(vi) CST element possesses
(a) Constant field variable throughout the element.
(b) Derivative of the field variable is constant throughout the element.
(c) Variation of the field variable is quadratic throughout the element.
(d) Variation of the field variable is cubic throughout the element.

(vii) The coefficient in stress-strain relation for a linear, elastic, isotropic material under plane strain condition is given by-

(a) $\frac{E}{(1+\nu)}\begin{bmatrix} 1-\nu & 0 & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$ (b) $\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$

(c) $\frac{E}{(1-2\nu)}\begin{bmatrix} 1-\nu & 0 & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$ (d) $\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} \nu & 0 & 0 \\ \nu & \nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$

(viii) For two-point numerical integration by Gauss Quadrature formula, the locations of sampling point (x_i) are

(a) $\left(+\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)$ (b) $\left(+\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{5}}\right)$ (c) $\left(+\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ (d) $\left(+\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(ix) Which one of the following software is not a FEA dedicated software

(a) AutoCAD (b) ANSYS (c) MSC Nastran-Patran (d) COMSOL

(x) The sequence of the numerical simulation in any FEA software is

(a) Pre-processing→Solution→Post-processing
(b) Post-processing→Solution→Pre-processing
(b) Pre-processing→Post-processing→Solution
(d) Solution→ Pre-processing→Post-processing

Fill in the blanks with the correct word

(xi) The Galerkin method is a type of _____ residual method.

(xii) For a 2-node BAR element of length 'L', the shape functions N_1 and N_2 in terms of domain variable 'x', are _____ and _____.

(xiii) If two spring element of stiffness 'K_1' and 'K_2' respectively are connected in series, then its global stiffness matrix of this spring combination would be _____.

(xiv) The coefficient in stress-strain relation for a linear, elastic, isotropic material having modulus of elasticity 'E' and Poisson's Ratio 'ν' under plane stress condition is _____.

(xv) Governing equations are often expressed as _____ equations.

Group - B

2. (a) What is Raileigh-Ritz Method? *[(CO2) (Understand/LOCQ)]*

(b) For the assembly of springs as shown Fig. 1 below, determine global stiffness matrix of the spring assembly using Rayleigh-Ritz Method. Also write down the final FEA formulation to find out deflections of each spring.

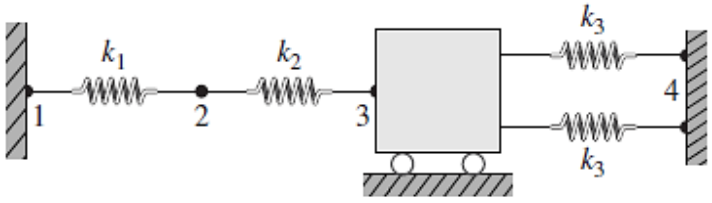

Fig. 1

[(CO2) (Analyse/IOCQ)]

2 + (7 + 3) = 12

3. (a) What do you understand by governing equation of any physical phenomenon?

[(CO1) (Understand/LOCQ)]

(b) Find the approximate solution of a uniform rod subjected to a uniform axial load has been shown in Fig. 2 below. The governing differential equation is given by:

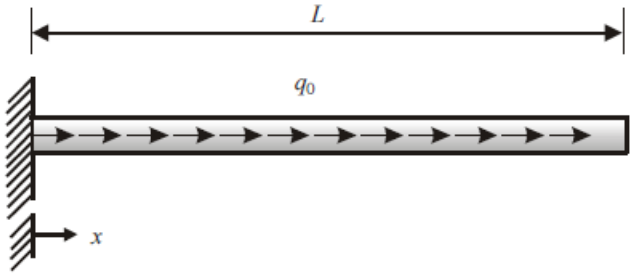

Fig. 2

$$AE\,\frac{\partial^2 u}{\partial x^2} + q_0 = 0$$

with the boundary conditions,

u(0) = 0 and $\frac{\partial u}{\partial x} = 0$ (at x=L)

By assuming the approximate function $u(x) \approx \hat{u}(x) = c_1x + c_2x^2$

[(CO1) (Analyse/IOCQ)]

2 + 10 = 12

Group - C

4. Fig. 3 depicts an assembly of two bar elements made of two different materials. Determine the nodal displacements, element stresses and reaction forces at support.

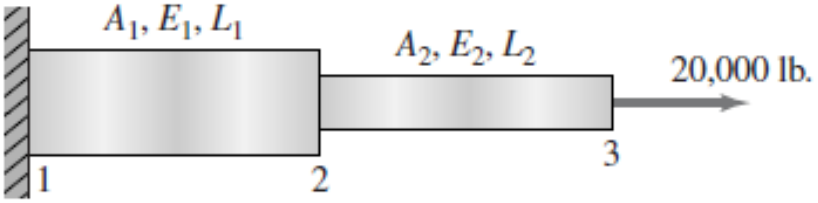

Fig. 3

Given $A_1 = 4$ in^2, $A_2 = 2.25$ in^2, $E_1 = 15 \times 10^6$ lb/in^2, $E_2 = 10 \times 10^6$ lb/in^2 and $L_1 = L_2 = 20$ in.

[(CO3) (Apply/LOCQ)]

(6 + 2 + 4) = 12

5. The plane truss shown in Fig. 4 below is subjected to a downward vertical load at joint 2. Determine the deflection of joint 2 in the global coordinate system, the axial stress in each element and Reactions at each support. For both elements A = 0.5 in^2 and E = 30 $\times 10^6$ psi.

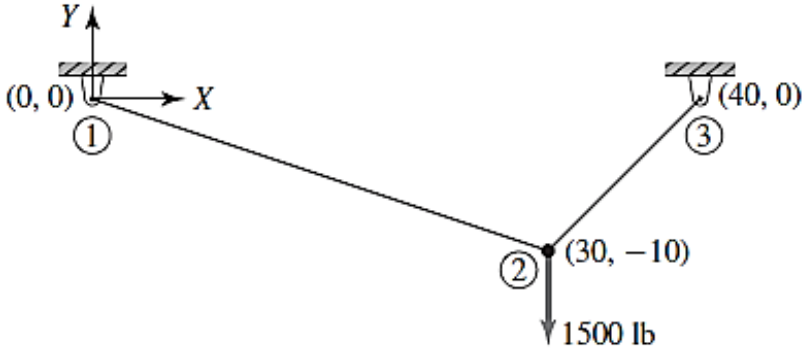

Fig. 4

[(CO3) (Apply/HOCQ)]

(5 + 4 + 3) = 12

Group - D

6. (a) Discuss in detail about Plane Stress and Axisymmetric condition for an object made of linear, elastic, isotropic material with suitable examples.
[(CO4) (Understand/IOCQ)]

(b) Mention constitutive relations for 'Plain Stress' and 'Axisymmetric' conditions.
[(CO4) (Remember/IOCQ)]

(3 + 3) + (3 + 3) = 12

7. (a) Schematically represent a Triangular element showing its nodal degree freedom in user coordinate system. *[(CO4) (Remember/LOCQ)]*

(b) Derive the expressions of shape functions of a Triangular element in user coordinate system. *[(CO4) (Understand/IOCQ)]*

(2 + 2) + 8 = 12

Group - E

8. Fig. 5 shows a quadrilateral element in user coordinate system. Show that the mapping correctly describes the line connecting nodes 2 and 3 and determine the (x, y) coordinates corresponding to the normalized coordinate $(\xi, \eta,)$= (0.6, 0.5).

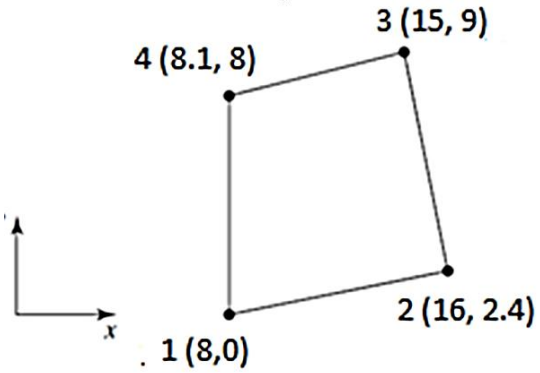

Fig. 5

[(CO5) (Analyse/HOCQ)]

(4 + 8) = 12

9. Evaluate the given integral analytically and numerically using Gauss–Legendre formula.

$$\int_{1}^{5} (x^3 + 6x^2 + 10)\, dx$$

Also compare the analytical and numerical results by calculating percentage error.
[(CO5) (Apply/IOCQ)]

(4 + 7 + 1) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	20.83	54.16	25.01