OPTIMIZATION TECHNIQUES
(MATH 6121)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The LPP Minimize $Z = x_1 + x_2$
Subject to
$$x_1 + 2x_2 \le 10$$
$$2x_1 + 2x_2 \ge 3$$
$$x_1, x_2 \ge 0$$
has
(a) a unique solution (b) no solution (c) infinitely many solutions (d) unbounded solution.

(ii) If the i-th constraint of the primal problem is an equality, then
(a) i-th variable of the dual is non negative in sign (b) i-th variable of the dual is negative in sign
(c) i-th variable of the dual is unrestricted in sign (d) i-th constraint of the dual is an equality.

(iii) If a constant be added to any row or any column of the cost matrix of an assignment problem, then
(a) the resulting assignment problem has the same optimal solution as the original problem
(b) the resulting assignment problem has a different optimal solution from the original problem
(c) the resulting assignment problem will not give an optimal solution
(d) the optimal solution of the resulting assignment problem can be obtained by adding same constant to the optimal solution of the original problem.

(iv) The degeneracy in the transportation problem indicates that
(a) dummy allocation needs to be added (b) solution is never optimal
(c) problem has to feasible solution (d) multiple optimal solution exists.

(v) If a function is convex on a convex domain then any local minimum is
(a) global maximum (b) global minimum
(c) local maximum (d) neither global maximum nor global minimum.

(vi) The range of p for which the following payoff matrix is strictly determinable is

PLAYER B

p	6	2	PLAYER A
-1	p	-7	
-2	4	p	

(a) $p \ge -1$ (b) $p \le 2$ (c) $-1 \le p \le 2$ (d) for any value of p.

(vii) The optimal strategy for player A for the following game problem is

PLAYER B

	10	2	3
PLAYER A	7	6	8
	0	3	1

(a) (0,0,1) (b) (0,1,0) (c) (1,0,0) (d) (0,0.5,0.5)

(viii) The quadratic form $Q(x, y, z) = x^2 - 2xy + y^2$ is
(a) positive definite (b) positive semi-definite
(c) negative definite (d) indefinite.

(ix) For the standard minimization problem, the Kuhn-Tucker necessary conditions are also sufficient conditions, provided
(a) objective function is convex and constraints are concave
(b) objective function and constraints are concave
(c) objective function and constraints are convex
(d) objective function is concave and constraints are convex.

(x) The bordered Hessian matrix of the Lagrange function L constructed for an optimization problem with equality constraints is given by

$$H^B(L) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 4 & 0 \\ 1 & 0 & 0 & 4 \end{pmatrix}$$

the number of variables involved in objective function are

(a) 1 (b) 2 (c) 3 (d) 0.

Fill in the blanks with the correct word

(xi) In Big-M method, for a maximization type problem, the coefficient of the artificial variable in the objective function is ___________.

(xii) If any of the constraints in the primal problem is a perfect equality, the corresponding dual variables is _____________ in sign.

(xiii) The number of basic variables in a transportation problem of size 7×8 is at most __________.

(xiv) Players apply mixed strategy in a game if there is no __________________.

(xv) The point of inflection occurs at $x = x_0$ for the function $f(x)$ provided $f^{(n)}(x_0) \neq 0$, for n ________.

Group - B

2. (a) Solve the following linear programming problem by graphical method:

Maximize $z = 5x_1 + 3x_2$
subject to the constraints

$$4x_1 - 3x_2 \leq 12$$
$$-x_1 + x_2 \geq -2$$
$$3x_1 + 2x_2 \geq 12$$
$$x_1 \geq 2,\ 0 \leq x_2 \leq 4.$$

[(MATH6121.1, MATH6121.2)(Apply/IOCQ)]

(b) Use simplex method to solve the following linear programming problem:

Maximize $z = 60x_1 + 50x_2$
subject to the constraints

$$x_1 + 2x_2 \leq 40$$
$$3x_1 + 2x_2 \leq 60$$
$$x_1,\ x_2 \geq 0.$$

[(MATH6121.1, MATH6121.2)(Apply/IOCQ)]

6 + 6 = 12

3. (a) Use Big-M method to solve the following linear programming problem:

Minimize $z = 12x_1 + 20x_2$
Subject to the constraints

$$x_1 + 8x_2 \geq 100$$
$$7x_1 + 12x_2 \geq 120$$
$$x_1,\ x_2 \geq 0.$$

[(MATH6121.1, MATH6121.2)(Apply/IOCQ)]

(b) Write the dual form of the given linear programming problem:

Maximize $z = 3x_1 + 4x_2 - x_3$
Subject to the constraints

$$x_1 + 2x_2 + x_3 \leq 5$$
$$-8x_1 + 6x_2 + x_3 = 19$$
$$3x_1 + x_2 - x_3 \geq 7$$

$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

[(MATH6121.1, MATH6121.2) (Understand/LOCQ)]

7 + 5 = 12

Group - C

4. (a) Solve the given transportation problem using VAM and find it's optimal solution:

	D_1	D_2	D_3	D_4	Availability
S_1	10	7	3	6	3
S_2	1	6	8	3	5
S_3	7	4	5	3	7
Demand	3	2	6	4	

[(MATH6121.1, MATH6121.2, MATH6121.3) (Evaluate/HOCQ)]

(b) Solve the following minimal assignment problem if there are four jobs $J_1,\ J_2,\ J_3,\ J_4$ and three machines A, B, C:

	A	B	C
J_1	9	26	15
J_2	13	27	6
J_3	35	20	15
J_4	18	30	20

[(MATH6121.1, MATH6121.2, MATH6121.3) (Apply/IOCQ)]

7 + 5 = 12

5. (a) Find the optimal assignment and minimum cost of the given assignment problem whose cost coefficients are given below:

	A	B	C	D	E
1	6	5	8	11	16
2	1	13	16	1	10
3	16	11	8	8	8
4	9	14	12	10	16
5	10	13	11	8	16

[(MATH6121.1MATH6121.2, MATH6121.3) (Apply/HOCQ)]

(b) Consider the following transportation problem and develop a linear programming (LP) model:

	D1	D2	D3	D4	Supply (Availability)
S1	23	27	16	18	30
S2	12	17	20	51	40
S3	22	28	12	32	53
Demand (Requirement)	22	35	25	41	

[(MATH6121.1, MATH6121.2, MATH6121.3) (Understand/LOCQ)]

7 + 5 = 12

Group - D

6. (a) Use graphical method in solving the following game and find the value of the game:

PLAYER B

PLAYER A

0	−2
7	−1
−1	4
−2	6
5	−3

[(MATH6121.1, MATH6121.4) (Understand/LOCQ)]

(b) Use dominance to reduce the following pay-off matrix to a 2×2 game and hence find the optimal strategies and the value of the game:

PLAYER B

PLAYER A

2	1	4	0
3	4	2	4
4	2	4	0
0	4	0	8

[(MATH6121.1, MATH6121.4) (Apply/IOCQ)]

6 + 6 = 12

7. (a) Use algebraic method to solve the following game:

PLAYER B

PLAYER A

9	1	4
0	6	3
5	2	8

[(MATH6121.1, MATH6121.4) (Apply/IOCQ)]

(b) In a rectangular game, the pay-off matrix is given by:

PLAYER B

PLAYER A

4	2	1	7	−1
1	−4	−6	−7	6
3	2	3	4	2
−6	1	−1	0	4
0	0	6	0	0

Find the optimal strategies and the value of the game.

[(MATH6121.1, MATH6121.4) (Understand/LOCQ)]

7 + 5 = 12

Group - E

8. (a) Use Kuhn-Tucker conditions to solve the following non-linear programming problem:

Maximize $Z = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$

Subject to the constraints

$x_1 + 3x_2 \leq 6$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0.$

[(MATH6121.5, MATH6121.6) (Evaluate/HOCQ)]

(b) Find the relative minimum or maximum (if any) of the function $f(x_1, x_2) = x_1 + 2x_2 + x_1x_2 - x_1^2 - x_2^2$.

[(MATH6121.5, MATH6121.6)(Analyse/IOCQ)]

8 + 4 = 12

9. Solve the following non-linear programming problem by the method of Lagrange's multiplier:

Minimize $Z = x_1^2 + (x_2 + 1)^2 + (x_3 - 1)^2$

Subject to the constraints

$x_1 + 5x_2 - 3x_3 = 6$

$x_1, x_2 \geq 0.$

[(MATH6121.5, MATH6121.6) (Evaluate/HOCQ)]

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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	21.88	42.7	35.42