METHODS IN OPTIMIZATION
(MATH 4121)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The feasible region of an L.P.P. is
(a) convex (b) non-convex (c) open (d) unbounded

(ii) The solution $x_1 = 1, x_2 = 2$ and $x_3 = 3$ of the system is
$$2x_1 + 2x_2 + 8x_3 = 30$$
$$x_1 + 4x_3 = 13$$
$$x_1 + 2x_2 + 4x_3 = 17$$
(a) a basic solution (b) a non-basic solution
(c) a degenerate basic feasible solution (d) a non-feasible basic solution

(iii) The optimal strategies for player A for the following game problem is

PLAYER *B*

PLAYER A	10	2	3
	7	6	8
	0	3	1

(a) (0,0,1) (b) (0,1,0) (c) $(\frac{1}{2}, \frac{1}{2}, 0)$ (d) (1,1,1)

(iv) In a fair game the value of the game is
(a) 1 (b) 0 (c) unbounded (d) 2 .

(v) In a transportation problem with m rows and n columns, $c_{ij} = u_i + v_j$ is related to
(a) basic cells (b) non basic cells
(c) all cells (d) cells of the last row.

(vi) For the standard maximization problem, the Kuhn-Tucker necessary conditions are also sufficient conditions, provided
(a) objective function is convex and constraints are concave
(b) objective function and constraints are concave
(c) objective function and constraints are convex
(d) objective function is concave and constraints are convex.

(vii) The quadratic form $Q(x,y,z) = x^2 + y^2 + 2xy + 2xz + 2yz + z^2$ is
(a) positive definite (b) positive semi definite
(c) negative definite (d) indefinite.

(viii) The reduction ratio $\frac{I_0}{I_n}$ for Fibonacci search algorithm where I_0 is the initial interval of uncertainty and I_n is the final interval of uncertainty is
(a) F_n (b) 1 (c) F_{n-1} (d) $(F_n)^2$.

(ix) A function is unimodal in $a \le x \le b$ if
(a) it contains a unique optimal solution (b) it contains two optimal solution
(c) it is a constant function (d) it is not a monotonic function.

(x) Which of the following is not a search algorithm?
(a) North-West Corner method (b) Interval halving method
(c) Fibonacci Search algorithm (d) Golden Section search algorithm.

Fill in the blanks with the correct word

(xi) If the primal problem has no feasible solution and the dual problem has a feasible solution, then we can conclude that the solution _______________.

(xii) A basic solution that satisfies the non-negativity condition is known as _________ solution.

(xiii) If a function is concave on a ___________ domain then any local maximum is global maximum.

(xiv) The value of the golden ratio is _____________.

(xv) The Fibonacci search algorithm is ___________ than the Interval halving method.

Group - B

2. (a) Egg contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and costs 12 paise per gram. Milk contains 8 units of vitamin A per gram and 12 units of vitamin B per gram and costs 20 paise per gram. The daily minimum requirements for vitamin A and vitamin B are 100 units and 120 units respectively. Find the minimum amount of eggs and milk required by formulating an L.P.P. and find the optimal solution by graphical method.
[(MATH4121.1, MATH4121.2) (Create/HOCQ)]

(b) Solve the following L.P.P. by Simplex method
Max $Z = 2x_1 - x_2 + 2x_3$,
subject to the constraints
$2x_1 + x_2 \le 10$,
$x_1 + 2x_2 - 2x_3 \le 20$,
$x_2 + 2x_3 \le 5$,
where $x_1, x_2, x_3 \ge 0$.
[(MATH4121.1, MATH4121.2)(Apply/IOCQ)]

5 + 7 = 12

3. (a) Use Big-M method to solve the following LPP

$Max\ Z = 2x_1 - 3x_2,$

Subject to the constraints

$-x_1 + x_2 \geq -2,$

$5x_1 + 4x_2 \leq 46$

$7x_1 + 2x_2 \geq 32$

where $x_1, x_2 \geq 0.$ *[(MATH4121.1, MATH4121.2)(Apply/IOCQ)]*

(b) Find the dual of the given L.P.P.:

Maximize $Z = 3x_1 + 4x_2$

subject to the constraints

$x_1 + 2x_2 \leq 5$

$-8x_1 + 6x_2 = 19$

$x_1,\ x_2 \geq 0$ *[(MATH4121.1, MATH4121.2) (Understand/LOCQ)]*

7 + 5 = 12

Group - C

4. (a) Use dominance to reduce the following pay-off matrix to a 2×2 game and hence find the optimal strategies and the value of the game:

PLAYER B

PLAYER A	3	5	4	2
	5	6	2	4
	2	1	4	0
	3	3	5	2

[(MATH4121.1, MATH4121.2, MATH4121.3, MATH4121.4) (Understand/LOCQ)]

(b) Use graphical method in solving the following game and find the value of the game. *[(MATH4121.1, MATH4121.2, MATH4121.3, MATH4121.4) (Apply/IOCQ)]*

PLAYER B

PLAYER A	0	-2
	7	-1
	-1	4
	-2	6
	5	-3

6 + 6 = 12

5. (a) Find the optimal basic feasible solution of the given Transportation problem:

	W_1	W_2	W_3	Supply
F_1	5	1	8	12
F_2	2	4	0	14
F_3	3	6	7	4
Demand	9	10	11	

[(MATH4121.1, MATH4121.2 MATH4121.3, MATH4121.4) (Evaluate/HOCQ)]

(b) Solve the assignment problem where the cost of assigning operators **I to IV** to machines **A to D** is given below:

	A	B	C	D
I	1	4	6	3
II	9	7	10	9
III	4	5	11	7
IV	8	7	8	5

[(MATH4121.1, MATH4121.2 MATH4121.3, MATH4121.4)(Evaluate/HOCQ)]

7 + 5 = 12

Group - D

6. (a) Verify whether the following function is convex or concave and find the maximum and minimum (if any)
$f(x_1, x_{2,} x_3) = x_1 + 2x_3 + x_2x_3 - x_1^2 - x_2^2 - x_3^2$. *[(MATH4121.6)(Remember/LOCQ)]*

(b) Use Kuhn-Tucker conditions to solve the following non-linear programming problem:
Maximize $z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$
subject to the constraints
$x_2 \leq 8$
$x_1 + x_2 \leq 10$
$x_1, x_2 \geq 0$ *[(MATH4121.6)(Evaluate/HOCQ)]*

4 + 8 = 12

7. (a) Solve the following non-linear programming problem by the method of Lagrange multiplier:
Minimize $Z = x_1^2 + (x_2 + 1)^2 + (x_3 - 1)^2$
subject to the constraint
$x_1 + 5x_2 - 3x_3 = 6$
$x_1, x_2, x_3 \geq 0$. *[(MATH4121.6)(Evaluate/HOCQ)]*

(b) Consider the function $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + x_3^2 + x_1x_2 - 2x_3 - 7x_1 + 12$. Find the local optimal point(s), if any, of the function. Determine also whether the local optima are global or not. *[(MATH4121.6)(Understand/LOCQ)]*

8 + 4 = 12

Group - E

8. Find the minimum of the function $f(x) = x^5 - 5x^3 - 20x + 5$ by interval halving method in the interval $[0, 5]$. Perform 6 iterations. *[(MATH4121.1, MATH4121.5)(Apply/IOCQ)]*

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9. Use Golden Section search algorithm to minimize $f(x) = x^2(x - 2.5)$ in $[0,\ 1]$ taking the stopping tolerance to be less than 0.2. *[(MATH4121.1, MATH4121.5)(Apply/IOCQ)]*

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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	19.79	45.83	34.38