# DIGITAL SIGNAL PROCESSING
## (ELEC 3141)

**Time Allotted : 2½ hrs** **Full Marks : 60**

***Figures out of the right margin indicate full marks.***

***Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.***

***Candidates are required to give answer in their own words as far as practicable.***

## Group – A

1. Answer any twelve: **12 × 1 = 12**

*Choose the correct alternative for the following*

(i) The sampling frequency of the signal $x(t) = 4\sin(200\pi t) + 2\cos(50\pi t)$ should be
(a) Greater or equal to 100 Hz (b) Greater or equal to 200 Hz
(c) Greater or equal to 50 Hz (d) Greater or equal to 250 Hz

(ii) The convolution between two sequence $x(n) = (3,\ 2,\ 4),\ -1 \le n \le 1$; and $h(n) = (-2,\ 3),\ 0 \le n \le 1$ is $y(n)$. The lower and upper time index interval for the output signal $y(n) = x(n) * h(n)$ is
(a) [0, 5] (b) [0, 2] (c) [-1, 2] (d) [-3, -2]

(iii) A sequence is said to be an even sequence if
(a) $x(n) = 2x(n)$ (b) $x(-n) = 2x(n)$
(c) $x(n) = x(n-1)$ (d) $x(-n) = x(n)$

(iv) The inverse z-transform of $\frac{3z}{z-2}, |z| > 2$ is
(a) $2(3)^n u(-n-1)$ (b) $3(2)^n u(-n-1)$
(c) $3(2)^n u(n)$ (d) $2(3)^n u(n)$

(v) The R.O.C of $z - transform$ of the discrete signal$x(n) = 3^n u(-n-1)$ (causal signal) is
(a) R.O.C: $|z| < 3$
(b) R.O.C: Complete $z - complex$ plane
(c) R.O.C= inside the unit circle of $z - plane$.
(d) R.O.C: $|z| > 3$

(vi) In impulse invariant transformation the digital frequency '$\omega$' for a given analog frequency '$\Omega$' is given by
(a) $\omega = \Omega T$ (b) $\omega = \frac{\Omega}{T}$ (c) $\omega = \frac{2\Omega}{T}$ (d) $\omega = \frac{T}{\Omega}$

(vii) If $\mathcal{F}\{x(n)\} = X(e^{j\omega})$ then $\mathcal{F}\{x(n-3)\}$ will be
(a) $e^{j3\omega}X(e^{-j\omega})$ (b) $e^{-j3\omega}X(e^{-j\omega})$
(c) $e^{-j3\omega}X(e^{j\omega})$ (d) $e^{j3\omega}X(e^{j\omega})$

(viii) The value of twiddle factor, $W_4^3$ is
(a) $j$ (b) $-j$ (c) 1 (d)$-1$

(ix) The structure that uses same delays for input and output samples is
(a) Direct form-I (b) Direct form-II
(c) Cascade (d) Parallel

(x) Impulse response of a LTI discrete time system (FIR-filter) described by $y(n) = 2x(n) + 6x(n-2) - 3x(n-4)$, where $x(n) = \delta(n)$ is
(a) $h(n) = (2, -3,\ 0, 6, -6);\ n = 0\ to\ 4$
(b) $h(n) = (0,\ 2,\ 6,\ 0, -3);\ n = 0\ to\ 4$
(c) $h(n) = (2,\ 6,\ 0,\ 0, -3);\ n = 0\ to\ 4$
(d) $h(n) = (2,\ 0, 6,\ 0, -3);\ n = 0\ to\ 4$

*Fill in the blanks with the correct word*

(xi) If the sampling frequency is less than twice of message signal frequency then _____ phenomenon happens.

(xii) The Z-transform of $nu(n)$ is _____.

(xiii) In a 32-point sequence, the total number of complex additions in DFT is _____.

(xiv) The transfer function of 2nd order normalised Butterworth filter is _____.

(xv) The direct form-I realisation of IIR system involves _____ additions.

## Group - B

2. (a) Evaluate the circular convolution of the following two sequences $h(n) = \{1, 2, 1, 4\}$ and $x(n) = \{5, 2, 3\}$. *[(CO1) (Evaluate /HOCQ)]*

(b) Examine whether the following signal $w(n)$ is an energy or a power signal. $w(n) = e^{4n}\,u(n)$ *[(CO1) (Analyse/IOCQ)]*

(c) Determine $g(n) = x\left(\frac{n}{2} - 1\right)$ and $z(n) = x\left(n - \frac{1}{2}\right)$, when $x(n) = \left\{1, \overset{\downarrow}{\underset{\uparrow}{2}}, 1, 5\right\}$. *[(CO1) (Analyse/IOCQ)]*

**4 + 3 + (2 + 3) = 12**

3. (a) Evaluate the output of the system whose impulse response is given by $h(n) = \left\{2, 5, \overset{\downarrow}{\underset{\uparrow}{1}}, 4\right\}$ for an input $x(n) = \left\{2, 4, \overset{\downarrow}{\underset{\uparrow}{1}}, 3\right\}$ by convolution technique. *[(CO1)(Evaluate/HOCQ)]*

(b) Examine whether the following system is a linear or not. $y(n) = 2x(n) + \frac{1}{x(n-2)}$ *[(CO1) (Analyse/IOCQ)]*

(c) Examine whether the following system is time invariant or not. $y(n) = x(-n-2)$ *[(CO1)(Analyse/IOCQ)]*

(d) Solve the difference equation described by,
$C(k+2) - 5C(k+1) + 2C(k) = u(k)$
Given that $C(0) = 0$, $C(1) = 2$. Consider $C(k)$ as output and $u(k)$ as input to the system which is a unit step signal. *[(CO1)(Analyse/IOCQ)]*

**4 + 2 + 2 + 4 = 12**

## Group - C

4. (a) Find the z-transform and ROC of $x(n) = (\frac{1}{3})^n u(n) + (-\frac{1}{4})^n u(-n)$. *[(CO2)(Analyze/IOCQ)]*

(b) Find the inverse z-transform of the function $X(z) = \frac{z^2+z}{(z-1)(z-3)}$, if ROC $|z| > 3$ *[(CO2)(Analyze/IOCQ)]*

(c) Determine the step response of the system whose input $(x(n))$ and output $(y(n))$ relationship is given by the difference equation $y(n) + \frac{1}{3}y(n-1) = x(n)$. *[(CO2)(Evaluate /HOCQ)]*

**4 + 4 + 4 =12**

5. (a) What is impulse invariant technique? Obtain the mapping between s-plane and z-plane for the impulse invariant transformation. *[(CO2)(Understand/LOCQ)]*

(b) For the analog transfer function $H(s) = \frac{2}{s^2+3s+2}$, determine the $H(z)$ using impulse invariant transformation if sampling time $T = 1$sec. *[(CO2)(Analyze/IOCQ)]*

(c) A digital filter with cut-off frequency 0.25π to be designed from the analog filter whose system response is $H(s) = \frac{\Omega_c}{s+\Omega_c}$. Use bilinear transformation to find the transfer function of the digital filter. *[(CO2)(Evaluate/HOCQ)]*

**4 + 4 + 4 =12**

## Group - D

6. (a) Determine the DTFT of the sequence $x(n) = \cos(\omega_0 n)$. *[(CO3)(Analyze/IOCQ)]*

(b) Evaluate 4-point DFT of the sequence $x(n) = (-1)^n$ using matrix method. *[(CO3)(Evaluate/HOCQ)]*

(c) Evaluate inverse DFT of $X(k) = \{1,0,1,0\}$. *[(CO3)(Evaluate/HOCQ)]*

**4 + 4 + 4 = 12**

7. (a) Given $x(n) = \cos(\frac{\pi n}{2})$ for $0 \le n \le 7$; and $x(n) = 0$ otherwise, compute 8-point DFT of $x(n)$ using DIT-FFT algorithm. *[(CO3)(Evaluate/HOCQ)]*

(b) Compare the number of multiplications and additions required to compute the DFT of a 1024-point sequence using direct computation and that using FFT. *[(CO3)(Understand/LOCQ)]*

**8 + 4 = 12**

## Group - E

8. Realize the direct form-I, direct form-II and cascade form of the system given by $H(z) = \frac{1+\frac{1}{2}z^{-1}+3z^{-2}}{1+\frac{3}{4}z^{-1}-\frac{1}{2}z^{-2}-\frac{1}{4}z^{-3}}$ *[(CO4)(Analyze/IOCQ)]*

**(4 + 4 + 4) = 12**

9. (a) What are FIR filters? Describe the advantages of using $FIR$ filters over the $IIR$ filters. *[(CO4)(Remember/LOCQ)]*

   (b) Design a linear phase FIR high pass filter with a cut-off frequency, $\omega_c = 0.7\pi$ rad/samples using 7 samples of Hamming window sequence. *[(CO4)(Evaluate/HOCQ)]*

**(2 + 2) + 8 = 12**

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| Cognition Level | LOCQ | IOCQ | HOCQ |
|---|---|---|---|
| Percentage distribution | 12.5 | 45.8 | 41.7 |