INFORMATION THEORY AND CODING
(CBS2102)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) Mutual information of the channel is
(a) H(A) + H(A/B) (b) H(A) / H(A/ B)
(c) H(A) - H(A/B) (d) H(A) . H(A/B)

(ii) If t is the error correction capability of BCH code, what is the minimum distance of the code?
(a) 2t (b) 2t + 1 (c) 2t-1 (d) None of these.

(iii) Which is not a field element of the polynomial, $p(x) = x^5 + x^2 + 1$ in GF (26)?
(a) $\alpha^3 + \alpha$ (b) $\alpha^4 + \alpha^2$ (c) $\alpha^4 + 1$ (d) $\alpha^3 + \alpha + 1$

(iv) The minimum distance (d_{min}) of a code is 5. How many errors can it correct?
(a) 1 (b) 2 (c) 3 (d) 4.

(v) What is the relationship between the generator polynomial and the parity check polynomial in cyclic codes?
(a) They are orthogonal to each other
(b) The generator polynomial is a multiple of the parity check polynomial
(c) The parity check polynomial is a multiple of the generator polynomial
(d) They are independent of each other.

(vi) A voice grade telephone channel has a bandwidth of 3400Hz. If the signal to noise ratio (SNR) on the channel is 30dB, the capacity of the channel is
(a) 35.88 kbits/ sec (b) 32.88 kbits/sec
(c) 34.88 kbits/sec (d) 33.88 kbits/sec

(vii) Which of the following is true about the Viterbi algorithm?
(a) It uses a brute-force search to decode convolutional codes.
(b) It is optimal for finding the most likely transmitted sequence given the received sequence.
(c) It relies on a lookup table of all possible codewords.
(d) It can only decode convolutional codes with fixed-length codewords.

(viii) The BCH code with parameters [15,7,5] means
(a) it has a length of 15 bits, 7 information bits, and can correct up to 5 errors
(b) it has a length of 15 bits, 7 parity bits, and a minimum distance of 5
(c) it can correct up to 5 errors and has a minimum distance of 7
(d) it has a length of 7 bits, 15 information bits, and can correct up to 5 errors.

(ix) Two discrete independent sources S1 and S2 have 8 and 16 equally likely messages respectively. Which of the following statement is correct if we compared the sources in terms of entropy?
(a) S1 = S2 (b) S1 < S2
(c) S1 ≥ S2 (d) Depends on rate of symbols.

(x) If R is a codeword and H is a parity check matrix, then which of the following is true for correctly received codeword?
(a) $RH = 0$ (b) $RH^t \neq 0$ (c) $R^tH = 0$ (d) $RH^t = 0$

Fill in the blanks with the correct word

(xi) If there are 'M' numbers of equally likely messages, the entropy of the source is _____.

(xii) Hamming weight of a code vector is the number of _____ components of Codeword.

(xiii) Encoder circuit of _____ is used a linear feed-forward shift register.

(xiv) A source emits 40 distinct symbols. The maximum entropy of the source is generated when probabilities are _____.

(xv) The binary symbols 0 & 1 are transmitted with probabilities 1/4 & 3/4 respectively. The corresponding self informations are _____.

Group - B

2. A discrete memory less source X has seven symbols x_1, x_2, x_3, x_4, x_5, x_6 and x_7 with probabilities $p(x_1) = 0.125$, $p(x_2) = 0.0625$, $p(x_3) = 0.25$, $p(x_4) = 0.0625$, $p(x_5) = 0.125$, $p(x_6) = 0.125$ and $p(x_7) = 0.25$. Encode each symbol of X using Huffman coding, and calculate the efficiency of the code. *[(CO2)(Apply/IOCQ)]*
12

3. (a) A discrete memory-less source emits six messages with probabilities {0.4, 0.2, 0.2,0.1, 0.1}. Apply the Shannon–Fano–Elias technique to encode the messages and determine its efficiency. *[(CO2)(Apply/IOCQ)]*
(b) What do you mean by code efficiency? *[(CO4)(Remember/LOCQ)]*
(c) How is mutual information related with relative entropy? *[(CO1) (Understand/LOCQ)]*
(d) If X and Y are discrete random sources and P(X,Y) is their joint probability distribution given as

$$P(X,Y) = \begin{pmatrix} 0.08 & 0.05 & 0.02 & 0.05 \\ 0.15 & 0.07 & 0.01 & 0.12 \\ 0.10 & 0.06 & 0.05 & 0.04 \\ 0.01 & 0.12 & 0.01 & 0.06 \end{pmatrix}$$

Calculate H(X), H(Y), H(X/Y), H(Y/X), H(X, Y) and I(X, Y). *[(CO2)(Apply/IOCQ)]*

4 + 2 + 2 + 4 = 12

Group - C

4. (a) Determine the encoded message for data message 100110111001 using CRC generator polynomial. *[(CO4)(Apply/IOCQ)]*

   (b) Prove that:
   (i) CHT = 0 where C is a valid code word and H is the parity check matrix.
   (ii) Syndrome is independent of the codeword. *[(CO4)(Analyse/HOCQ)]*

6 + (2 × 3) = 12

5. Consider a (8, 4) Hamming Code whose parity check equations are:
   $C_1 = m_1 \oplus m_2 \oplus m_3 \oplus m_4$
   $C_2 = m_1 \oplus m_3 \oplus m_4$
   $C_3 = m_1 \oplus m_4$
   $C_4 = m_2 \oplus m_3 \oplus m_4$
   Find out (i) Generator Matrix G
   (ii) Parity Check Matrix H
   (iii) d_{min} distance
   (iv) How many errors it can detect and correct? *[(CO4)(Apply/IOCQ)]*

(2 + 2 + 5 + 3) = 12

Group - D

6. (a) Determine the Galois Field elements of GF (24) for the corresponding polynomial $p(x) = x^4+x+1$. *[(CO6)(Apply/IOCQ)]*

   (b) What do you mean by primitive element? α3, α10 are field elements of GF(2^4), determine their order and check whether or not they are primitive elements. *[(CO6)(Apply/IOCQ)]*

   (c) Find the minimal polynomial of α4 in GF (2^4). *[(CO6)(Apply/IOCQ)]*

5 + 4 + 3 = 12

7. (a) Construct the field GF(2^5) for the given polynomial $p(x) = x^5 + x^2 + 1$. *[(CO6)(Apply/IOCQ)]*

   (b) Find (i) $\alpha^5 + \alpha^{12} + \alpha^{14}$ (ii) $\alpha^3 + \alpha^7 + \alpha^{11}$ (iii) $\alpha^{11} + \alpha^{13} + \alpha^2$ in GF (2^4). *[(CO6)(Apply/IOCQ)]*

   (c) Show that α^5 is a primitive element of GF(2^3). *[(CO6)(Analyse/HOCQ)]*

3 + 6 + 3 = 12

Group - E

8. (a) What do you mean by free distance? *[(CO5)(Remember/LOCQ)]*

   (b) Construct a (2, 1, 3) convolutional encoder with [1, 0, 1, 1] and [1, 1, 1, 1] as the impulse responses. Find the output of the convolutional encoder for input sequence 11011 using time and transform domain approach. *[(CO5)(Apply/IOCQ)]*

2 + (2 + 4 + 4) = 12

9. Consider a convolutional code with generator polynomials $g1(x)=1+x$ and $g2(x)=1+x^2$. The received sequence is (1,0,1,1,0,0,1).
   (i) Use the Viterbi algorithm to decode the received sequence.
   (ii) Construct the trellis diagram for the convolutional encoder.
   (iii) Show the computations of path metrics and how the most likely transmitted sequence is determined. *[(CO5)(Apply/IOCQ)]*

(4 + 4 + 4) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	6.25	87.09	9.38