B.TECH/CHE/5TH SEM/CHEN 3104/2016

NUMERICAL METHODS OF ANALYSIS (CHEN 3104)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The forward elimination step of Gauss Elimination process results in which kind of matrix?
(a) Lower Triangular (b) Upper Triangular
(c) Diagonally dominant (d) Identity.

(ii) Simpson's 3/8th rule is used in integration when
(a) the interval between data is non uniform
(b) data is spaced over equal segments
(c) a data point is missing
(d) none of the above.

(iii) The determinant of jacobian at x = 0, y = 0 of the following equations:
$x^2 - y + 1 = 0$
$Sin(x) - y = 0$
(a) is negative (b) > 2
(c) is positive (d) none of the above.

(iv) Floating point numbers can be represented with the following equation (m: mantissa, b: base of number system used, e: exponent)
(a) $e.m^b$ (b) $m.b^e$ (c) $m.e^b$ (d) $e.b^m$

(v) The truncation error for second order Taylor series approximation of $f(x) = x^2 - 3x$ is given by (h is difference interval in x)
(a) $2h^2$ (b) $-2 - h + 2h^2$ (c) $1 - 2h$ (d) $-2 - h$.

(vi) The truncation error, ε_t for $f(x) = Sin(\pi/3)$ on the basis of its derivatives at $\pi/4$ with the 1st order Taylor series approximation
(a) $1/(12\sqrt{2})$ (b) $13/(12\sqrt{2})$
(c) $1/12$ (d) $1/\sqrt{2}$.

(vii) Heun's Method of ODE integration is similar to
(a) 2nd order Runge Kutta
(b) 1st order Runge Kutta
(c) modified Euler method
(d) none of above.

(viii) Which of the following problems is encountered with Newton-Raphson root finding method?
(a) It is a linear method and hence less accurate
(b) Requires guessing an initial interval
(c) May give rise to divergent solution
(d) None of the above.

(ix) The half-bandwidth of a penta-diagonal system is
(a) 0 (b) 1 (c) 2 (d) none of the above.

(x) The minimum value of condition number of a matrix is
(a) 1 (b) 2 (c) 0 (d) -1.

Group – B

2. (a) A circular rod with an internal heat source S, with non-dimensional temperature at the ends is given by:

$$\frac{dT}{dr} = 0, \quad at \quad r = 0$$

$$T = 1, \quad at \quad r = 1$$

The governing equation for rod temperature is

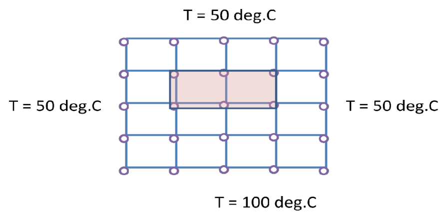
$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + S = 0$$

Assume that there are 2 interior points for finite differencing and the initial temperature at these points is zero. Derive the numerical difference equation for the given problem. If *r = 0.2 m*, S = $20K/m^2$, write out resulting linear system.

(b) Use an iterative method to solve the resulting system.

6 + 6 = 12

3. (a)



The figure above shows a slab, the bottom end of which is kept at 100^{0}C and the remaining three sides at 50^{0}C. The slab has a length of 8cm and a breadth of 4cm. The shaded area represents a heat sink, where q = -20 cal/cm^2/s. Write out the difference equations for temperature at nodes that are within the heat sink. The thermal diffusivity is given by 0.5 cm^2/s.

(b) How would you mathematically express the convergence criterion to obtain converged solutions of T?

10 + 2 = 12

Group – C

4. (a) Given the following set of data

x	1	2	3	5	7	8
f(x)	3	6	19	99	291	444

Calculate f(4) using Newton's interpolating polynomial of order 1 through 4.

(b) In (a), use lagrange interpolation to calculate $f(3.5)$.

6 + 6 = 12

5. (a) Use the fourth order RK method to solve over the range 0 to 0.5 using a step size of 0.1.

$$\frac{dy}{dx} = -2y + 4e^{-x}$$

$$\frac{dz}{dx} = -\frac{yz^2}{3}$$

$$y(0) = 2$$

$$z(0) = 4$$

(b) Give an example of a system that can generate an initial value ODE.

10 + 2 = 12

Group – D

6. (a) A tank containing liquid at height 7m drains through a pipe which is located at height e = 1m above the bottom of tank. The following differential equation states how the depth of the liquid in tank changes with time.

$$\frac{dh}{dt} = -\frac{\pi d^2}{4A(h)}\sqrt{2g(h+e)}$$

Where A(h) is related to h in the following manner

h	7	6	5	4	3	2	1
A(h)x10^4 m^2	1.5	1.17	0.97	0.67	0.45	0.32	0.18

Find the time taken for the height of the liquid to drop to the level of the drainage pipe. Time is measured in seconds. Diameter, d of pipe = 0.25m and h(0) = 6m, g = 9.81m/s^2.
Use appropriate integration method.

(b) Fit a lagrange polynomial through the given data of A(h) vs h. How else can you solve the above problem?

6 + (4 + 2) = 12

7. (a) Evaluate the following integral using Simpson's 1/3 rule:

$$\int_1^4 \sqrt{(1+x^3)}dx$$

(b) Solve the following system of equation using tridiagonal matrix algorithm;

$$3x_1 - x_2 = 4$$

$$-x_1 + 3x_2 - x_3 = 10$$

$$-2x_2 + 3x_3 = 5$$

6 + 6 = 12

Group – E

8. (a) Use LU decomposition to find the inverse of A matrix in Ax = b given below :

$$4x_1 + x_2 - x_3 = -2$$

$$5x_1 + x_2 + 2x_3 = 4$$

$$6x_1 + x_2 + x_3 = 6$$

(b) Interpolate between *log8 = 0.9030900* and *log12 = 1.0791812* using lagrange interpolation (CC 18.1).

8 + 4 = 12

9. (a) The rate of heat flow by conduction between two points on a cylinder heated at one end is given by

$$\frac{dQ}{dt} = \lambda A \frac{dT}{dx} \text{ and } \frac{dT}{dx} = \frac{100(L-x)(20-t)}{100-xt}$$

$$\textit{where } \lambda = 0.5 \textit{ cal cm/s and } A = 12cm^2$$

First combine the set of ODE to solve for the heat flow at specified times.

Then calculate the cumulative heat flow, Q from 0 to 20s. Given that at t = 0, Q(0) = 0, L = 20cm and x = 2.5cm. You may use Heun's method of integration.

(b) What is a stiff system? Give an example of a stiff system.

8 + 4 = 12