PROBABILITY AND STATISTICAL METHODS
(MTH2102)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively. Then $P(X=1)$ is equal to
(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{6}$ (d) $\frac{1}{32}$.

(ii) A random variable X has Poisson distribution such that $P(1)=P(2)$, then $P(0)=$
(a) $\frac{1}{e}$ (b) e^2 (c) $\frac{1}{e^2}$ (d) e^0.

(iii) For a random variable X with mean m and variance 2, the value of $P(|X-m| \leq 2) \geq ?$
(a) 0.9 (b) 0.5 (c) 0.2 (d) 0.3.

(iv) The joint probability density function of the random variable X and Y is given by $f(x,y)=\begin{cases} Ce^{-(x+y)}, 0<y<x<\infty \\ 0, \quad \text{otherwise} \end{cases}$, then the value of C is
(a) 1 (b) 2 (c) 5 (d) 7

(v) While calculating stationary probability of a Markov process, it is assumed that
(a) there is a single absorbing state
(b) transition probabilities do not change
(c) there is a single non-absorbing state
(d) none of the above.

(vi) If $x+4y+3=0$ and $4x+9y+5=0$ be the two regression lines, then the expectation of y is
(a) 1 (b) 2 (c) -1 (d) 0.

(vii) If two bivariates (x,y) and (u,v) are such that $x=-2u+4$ and $y=3v-6$, then
(a) $r_{xy}=r_{uv}$ (b) $r_{xy}=-6r_{uv}$ (c) $r_{xy}=-r_{uv}$ (d) $r_{xy}=6r_{uv}$.

(viii) If μ is a parameter and $H_0: \mu = 5$ is the null hypothesis, then which one of the following is left sided alternative hypothesis

(a) $H_1: \mu \neq 5$ (b) $H_1: \mu < 5$ (c) $H_1: \mu > 5$ (d) $H_1: \mu = 4$.

(ix) If $E(T_1) = \theta_1 + \theta_2$ and $E(T_2) = \theta_1 - \theta_2$ then the unbiased estimator of θ_1 is

(a) $T_1 + T_2$ (b) $\frac{1}{2}(T_1 - T_2)$ (c) $\frac{1}{2}(T_1 + T_2)$ (d) $T_1 - T_2$.

(x) In a test of hypothesis Type I error is committed when

(a) null hypothesis is rejected when it was really false
(b) null hypothesis is rejected when it was really true
(c) null hypothesis is accepted when it was really false
(d) null hypothesis is accepted when it was really true.

Fill in the blanks with the correct word

(xi) The moment generating function ($M_X(t)$) of Exponential distribution with mean $\frac{1}{2}$ is ____________.

(xii) The limiting distribution and stationary distribution of a transition probability matrix are equal if the underlying Markov chain is irreducible, positive recurrent and ____________.

(xiii) If $x + 5u = 2$, $2y + v = 7$ and the correlation coefficient of x and y is 0.25, then the correlation coefficient of u and v is ____________.

(xiv) If a population has Poisson distribution with parameter m, then the maximum likelihood estimate of m is ____________.

(xv) From a population with 8 members, a random sample (without replacement) of 2 members is drawn. The possible number of such samples (if order is ignored) is ____________.

Group - B

2. (a) A fair die is thrown 720 times. Use Chebyshev's inequality to obtain a lower bound for the probability of getting 91 to 149 sixes.

[(MTH2102.1,MTH2102.2)(Remember/LOCQ)]

(b) Suppose that the moment generating function of the random variable X is $M_X(t) = \left(\frac{1+3e^t}{4}\right)^{10}$. What is the mean and variance of X?

[(MTH2102.1,MTH2102.2)(Understand/LOCQ)]

(c) If the chance of being killed by flood during a year is 1/3000, use Poisson distribution to calculate the probability that out of 3000 persons living in a village, at least one will die in flood in a year. *[(MTH2102.1,MTH2102.2)(Apply/IOCQ)]*

4 + 4 + 4 = 12

3. (a) The lifetime of a certain brand of an electric bulb may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Using central limit theorem, determine the probability that the average lifetime of 60 bulbs exceed 1250 hours. *[(MTH2102.1,MTH2102.2)(Evaluate/HOCQ)]*

(b) The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund if it fails during the second year. If the manufacturer sells100 printers, how much should it expect to pay in refunds? *[(MTH2102.1,MTH2102.2)(Apply/IOCQ)]*

6 + 6 = 12

Group - C

4. (a) (i) State Chapman-Kolmogorov equation.

(ii) If the transition probability matrix of a Markov chain $\{X_n\}, n = 1,2,3 \ldots$ having three states $\{1,2,3\}$ is $P = \begin{pmatrix} 0.3 & 0.4 & 0.3 \\ 0.6 & 0.2 & 0.2 \\ 0.1 & 0.5 & 0.4 \end{pmatrix}$ and the initial distribution is $\Pi_0 = (0.2, 0.6, 0.2)$. Then find $P(X_2 = 3)$ and $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$. *[(MTH2102.1,MTH2102.2,MTH2102.3)(Apply/IOCQ)]*

(b) The two-dimensional random variables X and Y have a joint probability density function

$$f(x,y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find $P\left(X < \frac{1}{2}, Y < \frac{1}{4}\right)$.

(ii) Find all the marginal and conditional probability density functions.

[(MTH2102.1,MTH2102.2,MTH2102.3)(Remember/LOCQ)]

(2 + 4) + 6 = 12

5. (a) Following is the joint probability distribution of X and Y:

X \ Y	1	2	3
0	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{10}$
2	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{5}$

(i) Is it a valid distribution? Give reason.

(ii) Find the marginal probability mass function of X and Y.

(iii) Are X and Y independent? Justify. *[(MTH2102.1,MTH2102.2,MTH2102.3)(Remember/LOCQ)]*

(b) For the following transition probability matrix for states $\{0, 1, 2, 3\}$

$$\begin{bmatrix} 0 & 0.2 & 0.8 & 0 \\ 0.3 & 0.1 & 0 & 0.6 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(i) Draw the state transition diagram.

(ii) Identify the recurrent, transient and absorbing states, and the communicating classes. *[(MTH2102.1,MTH2102.2,MTH2102.3)(Remember/LOCQ)]*

6 + 6 = 12

Group - D

6. (a) Calculate the first four central moments and hence find the skewness and kurtosis for the following data:

Marks:	55 – 58	58 – 61	61 – 64	64 – 67	67 – 70
Frequency	12	17	23	18	11

Also comment on the shape of the distribution.

[(MTH2102.3,MTH2102.4,MTH2102.5,MTH2102.6)(Evaluate/HOCQ)]

(b) Out of the two regression lines given by $x + 4y + 3 = 0$ and $4x + 9y + 5 = 0$, which one is the regression line of "y on x" ? Justify your answer. Find the mean of x and mean of y. Find the correlation coefficient between x and y. Estimate also the value of x when $y = 1.3$. *[(MTH2102.3, MTH2102.4, MTH2102.5, MTH2102.6) (Analyze/IOCQ)]*

6 + 6 = 12

7. (a) Ten students obtained the following marks in Mathematics and Statistics. Calculate the Spearman's Rank Correlation coefficient between the marks of two subjects.

Student (Roll No)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Marks in Maths	78	36	98	25	75	82	90	62	65	39
Marks in Physics	84	51	91	60	68	62	86	58	53	47

[(MTH2102.3,MTH2102.4,MTH2102.5,MTH2102.6)(Evaluate/HOCQ)]

(b) Find the median and mode for the following frequency distribution:

Class Intervals:	0 – 4	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49
Frequency:	0	2	11	26	17	3	6	8	2	1

[(MTH2102.3,MTH2102.4,MTH2102.5,MTH2102.6)(Apply/IOCQ)]

6 + 6 = 12

Group - E

8. (a) A machine produces a component of a product with a standard deviation of 1.6 cm in length. A random sample of 64 components was selected from the output and this sample has a mean length of 90 cm. The customer will reject the part if it is either less than 88 cm or more than 92 cm. Does the 95% confidence interval for the true mean length of all the components produced ensure acceptance by the customer? *[(MTH2102.4,MTH2102.5,MTH2102.6)(Analyze/IOCQ)]*

(b) (i) If T is an unbiased estimator of the population parameter θ, then prove that $\sqrt{T}$ is biased estimator of $\sqrt{\theta}$.

(ii) Define consistent estimator. *[(MTH2102.4,MTH2102.5,MTH2102.6)(Remember/LOCQ)]*

6 + 6 = 12

9. (a) A sample of size $n = 100$ is drawn from a population having standard deviation $\sigma = 5.1$. Given that the sample mean is $\overline{x} = 21.6$. Find 95% confidence interval for the population mean μ. It is given that the area under the standard normal curve between $z = 0$ and $z = 1.96$ is 0.475. *[(MTH2102.4,MTH2102.5,MTH2102.6)(Apply/IOCQ)]*

(b) In order to test whether a coin is perfect, the coin is tossed 5 times. The null hypothesis of perfectness is rejected if more than 4 heads are obtained. What is the probability of Type I error? Find the probability of Type II error when the corresponding probability of head is 0.2. *[(MTH2102.4,MTH2102.5,MTH2102.6)(Analyze/IOCQ)]*

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	33.33	47.92	18.75