

DISCRETE MATHEMATICS
(MTH2103)

Time Allotted : 2½ hrs

Full Marks : 60

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
any 4 (four) from Group B to E, taking one from each group.*

Candidates are required to give answer in their own words as far as practicable.

Group – A


1. Answer any twelve:

12 × 1 = 12

Choose the correct alternative for the following

- (i) A complete graph with n vertices is planar if
(a) $n \geq 5$ (b) $n \leq 5$ (c) $n = 5$ (d) $n < 5$
- (ii) A connected planar graph has 5 regions and 10 edges. The number of vertices it has
(a) 7 (b) 8 (c) 10 (d) 5
- (iii) If a, b, c are positive integer such that $\gcd(a, b) = 1$ and $a|bc$, then
(a) $a|b$ (b) $b|c$ (c) $a|c$ (d) $c|a$
- (iv) If 2^{50} is divided by 7, the remainder is
(a) 1 (b) 2 (c) 3 (d) 4
- (v) One solution of linear congruence $6x \equiv 1 \pmod{13}$ is
(a) 6 (b) 2 (c) 11 (d) 0
- (vi) In a group of 13 children there must be at least x children who were born in the same month. The value of x is
(a) 1 (b) 3 (c) 2 (d) 4
- (vii) The sum of the co-efficient in the expression of $(x + y + z)^{10}$ is
(a) 2^{10} (b) 2^9 (c) 3^{10} (d) 3^9
- (viii) If p : 'Anil is rich' and q : 'Kanchan is poor' then the symbolic form of the statement 'Either Anil or Kanchan is rich' is
(a) $p \vee q$ (b) $p \vee \sim q$ (c) $\sim p \vee q$ (d) $\sim (p \wedge q)$
- (ix) The inverse of the statement formula $(\sim r \rightarrow s)$ is
(a) $(r \rightarrow \sim s)$ (b) $(\sim r \rightarrow \sim s)$ (c) $(s \rightarrow r)$ (d) $(s \rightarrow \sim r)$
- (x) Contrapositive of ' $\sim p \rightarrow \sim q$ ' is
(a) $p \rightarrow q$ (b) $q \rightarrow \sim p$ (c) $q \rightarrow p$ (d) $q \rightarrow \sim p$

Fill in the blanks with the correct word

- (xi) A statement T is called tautology if T is _____ for all possible values of its variable.
- (xii) The generating function of the infinite sequence 1, 1, 1, ... is _____.
- (xiii) Let p : “It is hot” and q : “It is raining”, then the symbolic form of the statement “It is hot or it is not raining” is _____.
- (xiv) The highest power of 3 which is contained in $15!$ is _____.
- (xv) The chromatic number of the graph  is _____.

Group - B

2. (a) Give an example of a graph which contains
 (i) an Eulerian circuit that is also a Hamiltonian circuit.
 (ii) an Eulerian circuit and a Hamiltonian circuit that are distinct.
 (iii) an Eulerian circuit but not a Hamiltonian circuit.
 (iv) a Hamiltonian circuit, but not an Eulerian circuit.
 (v) neither an Eulerian circuit nor a Hamiltonian circuit.
[(MTH2103.1,MTH2103.2)(Apply/IOCQ)]
- (b) Draw a planar graph representation of the graph K_4 . Justify your answer.
[(MTH2103.1, MTH2103.2)(Remember/LOCQ)]
- (c) Prove that $e \leq 2n - 4$ for a connected simple planar graph with $n(\geq 3)$ vertices and e edges.
[(MTH2103.1,MTH2103.2)(Apply/IOCQ)]
 $5 + 3 + 4 = 12$
3. (a) Find the chromatic polynomial of $K_{2,3}$. Hence find the chromatic number of $K_{2,3}$.
[(MTH2103.1,MTH2103.2)(Analyse/IOCQ)]
- (b) Prove that a complete bipartite graph $K_{m,n}$ is planar iff $m \leq 2$ or $n \leq 2$.
[(MTH2103.1,MTH2103.2)(Create/HOCQ)]
- (c) Sketch the 11 –vertex binary trees with minimum and maximum heights. Find also the path length of both the trees.
[(MTH2103.1,MTH2103.2)(Apply/IOCQ)]
 $6 + 4 + 2 = 12$

Group - C

4. (a) Prove by mathematical induction that
 $1.2 + 2.3 + 3.4 + 4.5 + 5.6 + \cdots n(n+1) = \frac{n(n+1)(n+2)}{3}$ for all $n \geq 1$.
[(MTH2103.3)(Analyse/IOCQ)]
- (b) Prove that, if a is an integer, then a^3 is of the form $9k$, $9k + 1$ or $9k + 8$, where k is an integer.
[(MTH2103.3)(Create/HOCQ)]
 $6 + 6 = 12$
5. (a) Find a particular solution and the general solution of the following Diophantine equation:
 $221x + 35y = 11$.
[(MTH2103.3)(Analyse/IOCQ)]

- (b) Solve the following simultaneous congruences using the Chinese Remainder Theorem:
- $$x \equiv 1(\text{mod } 3),$$
- $$x \equiv 2(\text{mod } 5),$$
- $$x \equiv 3(\text{mod } 7).$$

[(MTH2103.3)(Apply/IOCQ)]

6 + 6 = 12

Group - D

6. (a) A total of 1232 students have taken a course in Bengali, 879 have taken a course in English, and 114 have taken a course in Hindi. Further, 103 have taken courses in both Bengali and English, 23 have taken courses in both Bengali and Hindi, and 14 have taken courses in both English and Hindi. If 2092 students have taken at least one of Bengali, English and Hindi, how many students have taken a course in all three languages?

[(MTH2103.4)(Analyse/IOCQ)]

- (b) Solve the recurrence relation:

$$a_n = 5a_{n-1} - 6a_{n-2} + 3n, \text{ given that } a_0 = 0 \text{ and } a_1 = 1.$$

[(MTH2103.4)(Evaluate/HOCQ)]

6 + 6 = 12

7. (a) There are 3 piles of identical red, blue and green balls, where each pile contains at least 10 balls. In how many ways can 10 balls be selected
- if there is no restriction?
 - if at least one red ball must be selected?
 - if at least one red ball, at least 2 blue balls and at least 3 green balls must be selected?

[(MTH2103.4)(Analyse/IOCQ)]

- (b) Determine the number of integer solutions of the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 32$, where $x_i \geq 0, 1 \leq i \leq 4$.

[(MTH2103.4)(Create/HOCQ)]

6 + 6 = 12

Group - E

8. (a) Find a propositional formula that represents the following proposition:
"If I win the lottery or pass the examination, then I am happy."
- (b) Let p : 'He is intelligent' and q : 'He is fat' be two propositions. Write the following statement in logical form using the simple propositions p and q .
- He is fat but not intelligent.
 - It is not true that he is neither fat nor intelligent.
 - If he is intelligent then he is fat.
- (c) Construct the truth table for the following compound proposition:
 $((\sim p \rightarrow q) \wedge (r \rightarrow p)) \leftrightarrow (\sim q \vee r).$

[(MTH2103.5,MTH2103.6)(Analyse/IOCQ)]

[(MTH2103.5,MTH2103.6)(Understand/LOCQ)]

[(MTH2103.5,MTH2103.6)(Apply/IOCQ)]

3 + 3 + 6 = 12

9. (a) State the definitions of conjunctive normal form and disjunctive normal form.

[(MTH2103.5, MTH2103.6)(Understand/LOCQ)]

- (b) Determine the dual of the following statement:
 $p \wedge (q \vee r)$. *[(MTH2103.5,MTH2103.6)(Apply/IOCQ)]*
- (c) Find the disjunctive normal form (DNF) of the following statement:
 $\sim(\sim(p \leftrightarrow q) \wedge r)$. *[(MTH2103.5,MTH2103.6)(Analyse/IOCQ)]*
- 4 + 2 + 6 = 12**
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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	10.42	66.66	22.92