PHYSICS – I
(PHY1001)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) For a given central force $\vec{F} = F(r)\hat{r}$ which of the following statement is false?
(a) The orbit is confined in a plane (b) $\vec{r} \cdot \vec{F} = 0$
(c) $\vec{r} \times \vec{F} = 0$ (d) The angular momentum is conserved.

(ii) The shape of the orbit of a particle moving under the influence of central force when the $E = 0$ is
(a) hyperbola (b) parabola
(c) ellipse (d) straight line

(iii) A damped oscillator is acted upon by an external force $F = F_0 cos\omega t$. If ω_0 is the natural frequency of oscillation, the frequency of oscillation of the system at steady state is
(a) ω_0 (b) ω (c) $\omega - \omega_0$ (d) $\omega + \omega_0$

(iv) Damping constant γ resembles the dimension of
(a) length (b) mass (c) time (d) frequency

(v) The resonant frequency of an electrical LCR circuit is given by
(a) $\nu = 2\pi\sqrt{LC}$ (b) $\nu = \frac{1}{2\pi\sqrt{LC}}$
(c) $\nu = \frac{2\pi}{\sqrt{LC}}$ (d) $\nu = \frac{1}{2\pi}\sqrt{\frac{L}{C}}$

(vi) A light vector represented by $\vec{E}(x,t) = 2\cos(kx - \omega t)\hat{j}$ is
(a) circularly polarized (b) unpolarized
(c) elliptically polarized (d) plane polarized

(vii) When the liquid is introduced between the lens and the plate, the diameters of the rings
(a) decreases (b) increases
(c) remains same (d) none of these

(viii) Which of the following commutation relation may result to the uncertainty in measurement?

(a) $[\hat{x}, \hat{p}_x] = i\hbar$ (b) $[\hat{y}, \hat{p}_z] = 0$

(c) $[\hat{p}_y, \hat{p}_z] = 0$ (d) $[\hat{y}, \hat{z}] = 0$

(ix) The eigenvalue of a quantum observable is

(a) complex (b) real

(c) purely imaginary (d) bicomplex

(x) If ψ_1 and ψ_2 are two eigenstates of the Hamiltonian corresponding to two different eigenvalues. $c_1\psi_1 + c_2\psi_2$ is

(a) an eigen state (b) is not an eigenstate

(c) may be an eigenstate (d) may not be an eigenstate.

Fill in the blanks with the correct word

(xi) For a given central force $\vec{F} = Kr^{-2}\hat{r}$, angular momentum is _____.

(xii) If the orbit equation is given by $r = e^{b\theta}$, the force law is given by _____.

(xiii) The relation between relaxation time (τ) and logarithmic decrement (Δ) of a damped harmonic oscillator is _____.

(xiv) A quarter wave introduces a phase difference of _____.

(xv) $[H, A] = 0$ and$H\psi = E\psi$ implies $HA\psi =$ _____.

Group - B

2. (a) Starting from the differential equation of orbit, justify that in absence of any external force a planet will move in a straight line orbit. *[(CO1)(Evaluate/HOCQ)]*

(b) Obtain the expression for acceleration in plane polar co-ordinate system. *[(CO4)(Remember/LOCQ)]*

(c) Using definition of central force and Newton's laws of motion, establish that the motion of a planet is confined in a plane. *[(CO5)(Apply/IOCQ)]*

(d) The orbit of a particle under the influence of a central force is given by $r = e^{-\theta}$. Find out the corresponding force law. *[(CO5)(Remember/LOCQ)]*

3 + 3 + 3 + 3 = 12

3. (a) Show that for a particle moving in a central force field, the areal velocity is constant. *[(CO6)(Remember/LOCQ)]*

(b) Develop Kepler's 3rd law (law of periods) starting from the consideration of central force. *[(CO4)(Remember/LOCQ)]*

(c) How does the eccentricity of the orbit depend on the energy of the particle moving under the influence of inverse square central force? *[(CO6)(Analyse/HOCQ)]*

(d) Is the $\vec{F} = \cos\theta\,\hat{r}$ a central force? Explain using the help of definition of central force. *[(CO1)(Apply/IOCQ)]*

3 + 3 + 4 + 2 = 12

Group - C

4. (a) Starting from the consideration that the rate of loss of total energy of a damped harmonic oscillator is equal to the power dissipated by the damping force, find out the differential equation of motion of the damped pendulum.

[(CO3)(Analyse/HOCQ)]

(b) The displacement of a particle of mass 3 gm undergoing damped oscillation is given by

$x(t) = 10e^{-0.5t} \sin 3t$ cm.

(i) Find the velocity of the particle at t = π/6 sec

(ii) Find the logarithmic decrement and relaxation time of the system.

[(CO4)(Remember/LOCQ)]

(c) The damped motion of a particle is given by the following equation

$$4\frac{d^2x}{dt^2} + 6\pi\frac{dx}{dt} + \beta\pi^2 x = 0, \beta > 0$$

(i) Find the conditions for large, critical and weak damping in relation to the values of β.

(ii) If the motion starts from a displacement x(0) = -5 cm with a velocity v(0) = 3 cm/sec, comment on whether the particle will ever pass through the origin in a critically damped case. *[(CO2)(Apply/IOCQ)]*

3 + (2 + 2) + (3 + 2) = 12

5. (a) A simple harmonic oscillator subjected to a damping force (proportional to velocity) and an external periodic force ($F_0 \cos \omega t$) where the steady state solution of the forced vibration is given by

$$x_p(t) = \frac{f_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}}\left[\frac{\omega_0^2 - \omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}}cos\omega t + \frac{2\gamma\omega}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}}sin\omega t\right]$$

Where, $f_0 = \frac{F_0}{m}$, m=mass of the oscillator, ω_0 = natural angular frequency of the oscillator, γ= damping factor.

(i) Develop the relation of the phase angle between the displacement and the driving periodic force with the angular frequency of driving force.

(ii) Draw the plot of the velocity amplitude against the angular frequency of the driving force for two different values of damping factor.

(iii) Show that at velocity resonance the velocity is in phase with the driving force. *[(CO3)(Analyse/HOCQ)]*

(b) Examine that in the steady state, the time-averaged input power by the driving force equals the time-averaged power dissipated through the damping force.

[(CO4)(Remember/LOCQ)]

(3 + 2 + 3) + 4 = 12

Group - D

6. (a) What do you understand about an unpolarized light? What are the different ways to produce a polarized light from an unpolarized light? *[(CO2)(Analyse/HOCQ)]*

(b) Identify the state of polarization of a light wave vector represented by $\vec{E}(z,t) = \sin(kz - \omega t)\hat{j} + \sin(kz - \omega t)\hat{k}$. Illustrate using a representative diagram the nature of the polarized light wave vector. *[(CO4)(Remember/LOCQ)]*

(c) A beam of right circularly polarized light propagating along the Z-direction is allowed to pass through a QWP with Y-axis as the fast axis. Find the state of polarization of the emerging light. *[(CO4)(Apply/IOCQ)]*

(d) Calculate the thickness of a quarter wave plate (QWP) required for an operating wavelength $\lambda = 0.589\ nm$ when it is made of calcite with $\mu_e = 1.4864$ and $\mu_o = 1.66584$. *[(CO3)(Apply/IOCQ)]*

(1 + 2) + (3 + 1) + 3 + 2 = 12

7. (a) Obtain the condition for constructive and destructive interference for superposition of two light wave vectors. *[(CO3)(Analyse/HOCQ)]*

(b) Obtain the expression for the diameter of n^{th} dark ring in Newton's ring experiment. *[(CO4)(Remember/LOCQ)]*

(c) If the diameter of fourth in Newton's ring experiment is 2.52 mm and it is illuminated by a light of wavelength 6000 $\dot{A}$, then find out the radius of curvature of the plano-convex lens. *[(CO3)(Remember/LOCQ)]*

4 + 4 + 4 = 12

Group - E

8. A particle is confined in a box $0 \leq x \leq a$.

(i) Find the eigen-functions and energy eigenvalues corresponding to ground state and 1^{st} excited state.

(ii) Plot the above wave functions and respective probability densities as functions of x.

(iii) Test Heisenberg uncertainty principle in the 1st excited state. *[(CO4)(Create/HOCQ)]*

(4 + 4 + 4) = 12

9. (a) The state of a quantum system is given by $\psi = \frac{1}{\sqrt{3}}\psi_1 + \sqrt{\frac{2}{3}}\psi_2$, where, ψ_1 and ψ_2 are normalized eigenstates. Find the probability of finding the system in the respective eigenstates. *[(CO1)(Remember/LOCQ)]*

(b) Considering the unitary time evolution and Hamiltonian H as the generator of time evolution develop time dependent Schroedinger equation. *[(CO1)(Apply/IOCQ)]*

(c) Find the eigenvalues of the operator $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and eigen-vectors corresponding to them. *[(CO2)(Understand/LOCQ)]*

4 + 4 + 4 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	41.67	19.79	38.54