MATHEMATICS - I
(MTH1101)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) If $\lambda_1 = 2,\ \lambda_2 = -3$ and $\lambda_3 = 0$ be three eigenvalues of a 3×3 square matrix A, then the value of the determinant of A is
(a) -1 (b) -6 (c) 0 (d) 1.

(ii) If A is an orthogonal matrix then A^{-1} is
(a) symmetric (b) skew symmetric
(c) orthogonal (d) idempotent.

(iii) The series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots \infty$ is
(a) conditionally convergent (b) divergent
(c) oscillatory (d) absolutely convergent.

(iv) A vector normal to the plane $2x + 3y - z = 0$ is
(a) $2\hat{\imath} - 3\hat{\jmath} - \hat{k}$ (b) $2\hat{\imath} + 3\hat{\jmath} - \hat{k}$
(c) $-2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ (d) $2\hat{\imath} + 3\hat{\jmath} + \hat{k}$.

(v) The value of the constant a, so that the vector $\vec{f} = (x + 2y + az)\hat{\imath} + (2x - 3y - z)\hat{\jmath} + (4x - y + 2z)\,\hat{k}$ is irrotational, is
(a) 1 (b) 2 (c) 0 (d) 4.

(vi) If the differential equation $\left(y + \frac{1}{x} + \frac{1}{x^2 y}\right) dx + \left(x - \frac{1}{y} + \frac{A}{xy^2}\right) dy$ is exact then the value of A is
(a) x (b) 1 (c) $\frac{1}{xy}$ (d) $\frac{1}{y}$.

(vii) The order and degree of the differential equation $\left\{1 + \left(\frac{dy}{dx}\right)^4\right\}^{\frac{1}{3}} = \frac{d^2y}{dx^2}$ are
(a) 2, 4 (b) 2, 3 (c) 4, 3 (d) 4, 2.

(viii) The general solution of the differential equation $(y+x)dx + x\,dy = 0$ is

(a) $x^2 - y^2 = c$ (b) $x^2 - xy = c$

(c) $x^2 + 2xy = c$ (d) $x^2 - 2xy = c$

where c is an arbitrary constant.

(ix) The changed order of the integral $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx\,dy$ is

(a) $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} dy\,dx$ (b) $\int_0^{a} \int_0^{\sqrt{2ax-x^2}} dy\,dx$

(c) $\int_0^{a} \int_0^{-\sqrt{2ax-x^2}} dy\,dx$ (d) $\int_0^{2a} \int_0^{-\sqrt{2ax-x^2}} dy\,dx$

(x) If $x = u + v$ and $y = uv$, then $\frac{\partial(x,y)}{\partial(u,v)}$ is

(a) uv (b) $u - v$ (c) $u + v$ (d) $\frac{u}{v}$.

Fill in the blanks with the correct word

(xi) If λ is an eigenvalue of a non-singular matrix A, then $\frac{1}{\lambda}$ is an eigenvalue of _______________.

(xii) The rank of the matrix $A = \begin{pmatrix} 1 & 4 \\ 4 & 1 \end{pmatrix}$ is __________.

(xiii) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then $div\,\vec{r}$ is equal to ______________.

(xiv) The value of the line integral $\int_C (dx - xdy)$ is where C is the line joining $(0,1)$ to $(1,0)$ is____________________.

(xv) Integrating factor of $\frac{dx}{dy} + \frac{x}{y\log y} = \frac{2}{y}$ is ____________________.

Group - B

2. (a) Reduce the following matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ to a row reduced echelon form and hence find its rank. *[(MTH1101.1, MTH1101.2) (Understand /LOCQ)]*

(b) State Cayley-Hamilton theorem and verify the theorem for the matrix

$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{bmatrix}$. *[(MTH1101.1, MTH1101.2 (Apply /IOCQ)]*

6 + 6 = 12

3. (a) If $\lambda \neq -14$, then show that the system of equations

$5x + 2y - z = 1$

$2x + 3y + 4z = 7$

$4x - 5y + \lambda z = \lambda - 5$ has a unique solution $(0,1,1)$.

[(MTH1101.1, MTH1101.2)(Understand /LOCQ)]

(b) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 7 \\ 4 & 0 & 3 \end{bmatrix}$.

[(MTH1101.1, MTH1101.2) (Understand /LOCQ)]

6 + 6 = 12

Group - C

4. (a) If $r = |\vec{r}|$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, rove that

(i) $\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$ and (ii) $\vec{\nabla}(r^n) = n\, r^{n-2}\vec{r}$.

[(MTH1101.3, MTH1101.4)(Understand/LOCQ)]

(b) Discuss the convergence of the series

$1 + \frac{(1!)^2}{2!}x + \frac{(2!)^2}{4!}x^2 + \frac{(3!)^2}{6!}x^3 + \cdots \infty, x > 0.$ *[(MTH1101.3, MTH1101.4)(Analyse/IOCQ)]*

6 + 6 = 12

5. (a) Show that $\left\{\frac{3n+1}{n+2}\right\}, n \in N$ is a bounded sequence.

[(MTH1101.3, MTH1101.4) (Remember/LOCQ)]

(b) Examine the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1} - \sqrt{n^4-1}\right)$.

[(MTH1101.3, MTH1101.4) (Analyse/IOCQ)]

(c) A function $\varphi(x,y,z)$ is such that $\vec{\nabla}\varphi = 2xyz^3\hat{\imath} + x^2z^3\hat{\jmath} + 3x^2yz^2\hat{k}$. If $\varphi(1,-2,2) = 4$, find $\varphi(x,y,z)$. *[(MTH1101.3, MTH1101.4)(Evaluate/HOCQ)]*

2 + 4 + 6 = 12

Group - D

6. (a) Find the singular and general solutions for the equation $y = px + p^2$ where $p \equiv \frac{dy}{dx}$.

[(MTH1101.5)(Evaluate/HOCQ)]

(b) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$.

[(MTH1101.5)(Apply/IOCQ)]

6 + 6 = 12

7. (a) Check whether the following equation is exact or not and then solve it.

$(xy^3 + y)dx + 2\,(x^2y^2 + x + y^4)dy = 0$ *[(MTH1101.5)(Understand/LOCQ)]*

(b) Solve the following Cauchy-Euler equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = x\,sin(logx)$.

[(MTH1101.5)(Apply/IOCQ)]

6 + 6 = 12

Group - E

8. (a) Change the order of the integration and hence evaluate $\int_0^1\int_{e^x}^{e}\frac{dx\,dy}{y^2\log y}$.

[(MTH1101.6)(Analyse/IOCQ)]

(b) Applying Euler's theorem on homogeneous function, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$, where

$u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$. Hence show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$.

[(MTH1101.6)(Remember/LOCQ)]

5 + 7 = 12

9. (a) Evaluate by Green's theorem $\int_\Gamma\{(2xy - x^2)dx + (x + y^2)dy\}$ where Γ is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$.

[(MTH1101.6) (Evaluate/HOCQ)]

(b) If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$.

[(MTH1101.6) (Understand/LOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	46.87	34.38	18.75