PHYSICS II (AEIE)
(PHY2102)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The integral $\oint \frac{dz}{z-a}$ over the circumference of the circle $|z-a| = R$ is equal to
(a) 2πi (b) πi (c) 0 (d) 1

(ii) The characteristics of classical $(1+1)$ dimensional classical wave equation $\psi_{xx} - v^{-2}\psi_{tt} = 0$are given by
(a) $x \mp vt = constant$ (b) $x^2 + vt = const$
(c) $x^2 + vt^2 = const$ (d) $4x^2vt^2 = const$

(iii) Drift velocity for unit electric field of a metal is called
(a) electrical conductivity (b) thermal conductivity
(c) electrical polarisation (d) mobility of charge carriers

(iv) Drift velocity of an electron in metal is given by
(a) $v_d = \frac{mE}{e\tau}$ (b) $v_d = \frac{e\tau}{m}E$
(c) $v_d = -\frac{e\tau}{m}E$ (d) $v_d = -\frac{e\tau}{mE}$

(v) Bloch theorem is applicable to
(a) periodic potential (b) constant potential
(c) infinite potential (d) zero potential

(vi) The Lorentz force acting on a unit charge moving with a velocity $5\hat{\imath} + 6\hat{\jmath}$ in a magnetic field $\vec{B} = 2\hat{\imath} + 5\hat{k}$ is
(a) $30\hat{\imath} - 12\hat{k}$ (b) $30\hat{\imath} - 25\hat{\jmath} - 25\hat{k}$
(c) $25\hat{\imath} - 12\hat{\jmath}$ (d) $30\hat{\imath} - 25\hat{\jmath} - 12\hat{k}$

(vii) Differential form of Faraday's law is
(a) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (b) $\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$
(c) $\vec{\nabla} \times \vec{E} = -\vec{B}$ (d) $\vec{\nabla} \times \vec{E} = \vec{B}$

(viii) Given the current density $\vec{J}$, electric field $\vec{E}$ and conductivity σ the Ohm's law is given by

(a) $\vec{E} = \sigma\vec{J}$ (b) $\sigma\vec{E} = \vec{J}$

(c) $\vec{E}.\vec{j} = \sigma$ (d) $\vec{E} \times \vec{J} \neq 0$

(ix) At nanoscale, the surface-to-volume ratio is

(a) very high (b) very low

(c) 1:1 (d) 1:5

(x) In vacuum, electric susceptibility is

(a) greater than 1 (b) less than 1

(c) zero (d) none of (a), (b) & (c).

Fill in the blanks with the correct word

(xi) The time at which drift velocity decays to (1/e) times its initial value is called as _____.

(xii) The region of K for which first Brillouin Zone exists is _____ to _____.

(xiii) For an electromagnetic wave in vacuum the electric field, and magnetic fields are both perpendicular to _____ vector.

(xiv) Fourier transform of $f(x-a)$ is equal to Fourier Transform of $f(x)$ multiplied by _____.

(xv) An electrostatic field has curl equals _____.

Group - B

2. (a) Categorize the types of singularity of the functions $f(x) = \frac{\sin z}{z}$ and $f(x) = \frac{\cos z}{z}$. *[(CO3)(Analyse/IOCQ)]*

(b) Determine the poles of the function $f(z) = \frac{1}{z^2+1}$ find the residues at the respective poles. *[(CO4)(Evaluate/HOCQ)]*

(c) Use argument principle to evaluate the integral $\oint \frac{2z+5}{z^2+5z+6} dz$. *[(CO2)(Evaluate/HOCQ)]*

4 + 4 + 4 = 12

3. (a) Prove the Parseval identity $\int_{-\infty}^{\infty} f(x)\overline{g(x)}dx = \int_{-\infty}^{\infty} F(k)\overline{G(k)}dk$ and hence prove $\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(k)|^2 dk$. *[(CO3)(Evaluate/HOCQ)]*

(b) Solve the following coupled differential equation using Laplace Transform: $\frac{dx}{dt} - y = 0$ and $\frac{dy}{dt} + 2x - 3y = 0$ with the initial conditions $x(0) = 0$ and $y(0) = 1$. *[(CO4)(Create/HOCQ)]*

(5 + 2) + 5 = 12

Group - C

4. (a) Assuming that each aluminium atom contributes three free electrons for conduction. The relevant data is given below.

Resistivity of aluminium ρ=2.7×10^{-8}Ω cm, Atomic weight M_A=26.98, density, ρ_s=2.7×10^{3}Kg/m^3, Avogadro number N_A=6.025×10^{23}.

Calculate

(i) Free electron concentration

(ii) Mobility

(iii) Drift velocity of electrons in aluminium wire of length 5 cm and resistance 0.06 Ω carrying current of 15 A. *[(CO2)(Evaluate/IOCQ)]*

(b) Starting from Schrodinger equation find wave function and energy of a free electron confined in a cubical box of length a. *[(CO2)(Analyse/IOCQ)]*

6 + (3 + 3) = 12

5. (a) From the expression of $P\frac{\sin\alpha a}{\alpha a}+\cos\alpha a=\cos ka$.

Find the lowest energy band for $P<<1$. *[(CO2)(Apply/IOCQ)]*

(b) The energy wave vector dispersion relation for a one dimensional crystal of lattice constant a is given by $E(k)=E_0-\alpha-2\beta\cos ka$ where E_0,α,β are constant. Obtain the effective mass of electron at the bottom and at the top of the band. *[(CO2)(Apply/IOCQ)]*

(c) What are Brillouin zones? *[(CO2)(Understand/IOCQ)]*

6 + 4 + 2 = 12

Group - D

6. (a) The electric field of an electromagnetic wave is polarized along the direction $\hat{\imath}+2\hat{\jmath}$. It is moving along the direction $2\hat{\imath}-\hat{\jmath}$. Find the equation of the plane of vibration find the value of the wave function at $\vec{r}=2\hat{\imath}+3\hat{\jmath}-\hat{k}$ if it is given by $\vec{E}_0\cos(\vec{k}.\vec{r}-\omega t)$ for $|\vec{E}|=5, |\vec{k}|=3$ and $\omega=60$. *[(CO3)(Remember/LOCQ)]*

(b) Calculate the magnetic field corresponding to the above electric field. *[(CO4)(Remember/LOCQ)]*

(c) Write the integral form of Faraday-Lenz law and find their differential expression. *[(CO2)(Remember/LOCQ)]*

6 + 3 + 3 = 12

7. (a) A conductor is identified by the Ohm's law $\vec{J}=\sigma\vec{E}$. Establish the related wave equations of Electromagnetic field for a conductor. *[(CO3)(Analyse/IOCQ)]*

(b) If an electromagnetic plane wave has the expression $\vec{E}(x,t)=\vec{E}_0e^{i(kx-\omega t)}$ and $\vec{B}(x,t)=\vec{B}_0e^{i(kx-\omega t)}$ show that $k^2=\mu\varepsilon\omega^2+i\mu\sigma\omega$ and hence find the possible values of k. *[(CO4)(Remember/LOCQ)]*

(c) When the electric field is along the plane of incidence the ratio of the amplitudes of the reflected ray and the incident ray is given by $\frac{E_R}{EI}=\frac{\tan(\theta_T-\theta_I)}{\tan(\theta_T+\theta_I)}$. Conclude Brewster Law from this result. *[(CO2)(Analyse/IOCQ)]*

4 + 4 + (3 + 1) = 12

Group - E

8. (a) What is effective mass of an electron? *[(CO4)(Understand/IOCQ)]*
Show that when an electron moves through a crystal the effective mass of the electron can be expressed as $m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}$ where symbols have their usual meaning. *[(CO4)(Analyse/IOCQ)]*

(b) Discuss the variation of position of Fermi level in extrinsic semiconductor with temperature. *[(CO4)(Understand/IOCQ)]*

(c) Explain graphically how electrical conductivity of an n-type semiconductor varies with temperature. *[(CO4)(Understand/IOCQ)]*

(2 + 4) + 3 + 3 = 12

9. (a) Define the following terms:
(i) Dipole moment, (ii) Electrical Susceptibility, (iii) Relative dielectric constant (iv) polarisation. *[(CO4)(Understand/IOCQ)]*

(b) A capacitor uses a dielctric material of relative permittivity ε_r = 8. It has an effective surface area of 0.036 m^2 with a capacitance of 6 μF. Calculate the field strength and dipole moment per unit volume if a potential difference of 15 V exists across the capacitor. *[(CO4)(Understand/IOCQ)]*

(4 × 2) + 4 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	16.67	62.50	20.83