M.TECH/AEIE/1ST SEM/MATH 5101/2016

ADVANCED MATHEMATICAL METHODS
(MATH 5101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The set S = {(x, y): $x^2 + y^2 = 25$} is
(a) convex (b) non-convex
(c) concave (d) none of these.

(ii) Dual of the Dual of an L.P.P is
(a) dual (b) primal
(c) another LPP (d) none of these.

(iii) In general K.K.T conditions for a non linear programming problem is
(a) necessary (b) sufficient
(c) necessary & sufficient (d) none of these.

(iv) The duality gap in LPP is
(a) 1 (b) infinity
(c) 0 (d) negative.

(v) The sum of Eigen values of $\begin{pmatrix} -1 & -2 & -1 \\ -2 & 3 & 2 \\ -1 & 2 & -3 \end{pmatrix}$ is
(a) -3 (b) -1 (c) 3 (d) 1.

(vi) If A and B are non-zero square matrices, then AB = 0 implies
(a) A and B are orthogonal (b) A and B are singular
(c) B is singular (d) A is singular.

(vii) Let U and W be subspaces of a vector space V over a field F. Then linear sum U + W
(a) is not a subspace of V (b) is the smallest subspace of V
(c) is a subspace but not smallest (d) none of these.

(viii) The system of equations x + 2y = 5, 2x + 4y =7 has
(a) unique solution (b) no solution
(c) infinite number of solutions (d) none of these.

(ix) The number of variables in the dual of an LPP is equal to
(a) the number of variables in the primal LPP
(b) the number of constraints in the primal LPP
(c) the number of constraints in the dual LPP
(d) none of the above.

(x) The critical point of $f(x,y) = x^2 + y^2$ is
(a) (0,0) (b) (1,1) (c) (2,2) (d) (3,3).

Group – B

2. (a) If V be the vector space of functions $f : R \to R$, show that W is a subspace of V where
(i) $W = \{f : f(1) = 0\}$ (ii) $W = \{f : f(3) = f(1)\}$

(b) If u = (1,3,-4,2), v = (4,-2,2,1), w = (5,-1,-2,6) in R^4, then show that
(i) $\langle 3u - 2v, w\rangle = 3\langle u, w\rangle - 2\langle v, w\rangle$ and (ii) normalize u and v.

(3 + 3) + (3 + 3) = 12

3. (a) Use Gram – Schmidt process to obtain an orthogonal basis of the subspace of the Euclidean space R^4 generated by the linearly independent set
{(1, 1, 0, 1), (1, 1, 0, 0), (0, 1, 0, 1)}.

(b) Let V be a real Vector space with { α, β, γ } as a basis. Prove that the set { α+ β + γ, β + γ, γ } is also a basis of V.

6 + 6 = 12

Group – C

4. (a) If F: $R^2 \to R^2$ defined by F(x, y) = (xy, x) then show that F is not linear.

(b) Suppose a linear mapping F: V → U is one-to-one and onto. Show that the inverse mapping F^{-1}: U → V is also linear.

6 + 6 = 12

5. (a) Let $A = \begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$. Find all the eigen values and the corresponding eigen vectors of A.

(b) If λ be an eigen value of a linear operator $T : V \to V$ and E_λ consists of all the eigen vectors belonging to λ, prove that E_λ is a subspace of V.

6 + 6 = 12

Group – D

6. (a) Use the method of Lagrange multiplier to maximize x^3y^5 subject to the constraint $x + y = 8$

(b) Find the stationary points and their nature for the following function:
$F(x,y) = 2x^3 + 6xy^2 - 3y^3 - 150x$

6 + 6 = 12

7. Solve the following optimization problem using Kuhn-Tucker conditions:
Minimize $x_1^2+2x_2^2+3x_3^2$
Subject to $x_1-x_2-2x_3 \le 12$
$x_1+2x_2-3x_3 \le 8$

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Group – E

8. (a) Solve by Simplex Method:
Maximize $z = x_1 - x_2 + 3x_3$
Subject to $x_1 + x_2 + x_3 \le 10$
$2x_1 - x_3 \le 2$
$2x_1 - 2x_2 + 3x_3 \le 0$
$x_1, x_2, x_3 \ge 0.$

(b) Write down the dual of the following optimization problem:
Maximize $z = 2x_1 - 3x_2$
Subject to $x_1 - 4x_2 \le 10$
$-x_1 + x_2 \le 3$
$-x_1 + 3x_2 \ge 4$
$x_1, x_2 \ge 0.$

8 + 4 = 12

9. Use Big-M method to solve the following LPP:
Minimize $12x+20y$
Subject to $6x+8y \ge 100$
$7x+12y \ge 120$
$x, y \ge 0.$

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