

**DIGITAL SIGNALS AND SYSTEMS
(AEIE 5102)**

Time Allotted : 3 hrs

Full Marks : 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

**Group - A
(Multiple Choice Type Questions)**

1. Choose the correct alternative for the following: **10 × 1 = 10**
- (i) The process of conversion of continuous time signal into discrete time signal is known as
 (a) aliasing (b) sampling
 (c) convolution (d) none of the above.
- (ii) In a signal $x(n)$, if 'n' is replaced by $2n$, then it is called
 (a) upsampling (b) folded version
 (c) downsampling (d) shifted version.
- (iii) The discrete time system, $y(n) = 0.7x(n-4) - 1.2x(n-5)$ is a,
 (a) dynamic system (b) memoryless system
 (c) time varying system (d) none of the above.
- (iv) If F_s is sampling frequency then the relation between analog frequency F and digital frequency f signal is,
 (a) $f = \frac{F}{2F_s}$ (b) $f = \frac{F_s}{F}$ (c) $f = \frac{F}{F_s}$ (d) $f = \frac{2F}{F_s}$
- (v) The complex valued phase factor/twiddle factor, W_N can be expressed as
 (a) $e^{-j2\pi n}$ (b) $e^{-\frac{j2\pi}{N}}$
 (c) $e^{-j2\pi}$ (d) $e^{-j2\pi kn}$

- (vi) In an N -point sequence, if $N = 16$, the total number of complex additions and multiplications using Radix-2 FFT are
 (a) 64 and 80 (b) 64 and 32
 (c) 80 and 64 (d) 24 and 12
- (vii) The inverse DFT of $x(n)$ is given by
 (a) $x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{-\frac{j2\pi kn}{N}}$ (b) $x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{\frac{j2\pi kn}{N}}$
 (c) $x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(n)e^{-\frac{j2\pi kn}{N}}$ (d) $x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{-\frac{j2\pi kn}{N}}$
- (viii) The linear phase realization structure is used to represent
 (a) FIR systems (b) IIR systems
 (c) both FIR and IIR systems (d) all discrete time systems.
- (ix) In practice, the zero-valued samples inserted by the up-sampler are replaced with appropriate nonzero values using some type of filtering process, the process is called
 (a) interpolation (b) decimation
 (c) both (a) and (b) (d) none of above.
- (x) If $\text{var}\{x\}$ and $\text{var}\{y\}$ are variances of two random uncorrelated signals x and y , respectively, then variance $\text{var}\{x+y\}$ of sum of the two signals is given by
 (a) $\text{var}\{x+y\} = \text{var}\{x\} + \text{var}\{y\}$ (b) $\text{var}\{x+y\} = \text{var}\{x\} - \text{var}\{y\}$
 (c) $\text{var}\{x+y\} = \text{var}\{x\} \times \text{var}\{y\}$ (d) $\text{var}\{x+y\} = \text{var}\{x\} / \text{var}\{y\}$

Group - B

2. (a) What are causal and non-causal signals?
 (b) Determine the even and odd part of the signal $x(n) = a^n$.
 (c) An analog signal is given by $x_a(t) = 10 \cos 100\pi t$. If the sampling frequency is 75Hz, find the discrete time signal $x(n)$. Also find an alias frequency corresponding to $F_s = 75\text{Hz}$.

2 + 3 + 7 = 12

3. (a) What do you mean by a linear and nonlinear system? Test the linearity of the system $y(n) = x^2(n)$.
- (b) Perform circular convolution of the two sequences $x_1(n) = \{2, 1, 2, 1\}$ and $x_2(n) = \{1, 2, 3, 4\}$.
- (2 + 5) + 5 = 12**

Group - C

4. (a) If $DFT \{x(n)\} = X(k)$, then prove that $DFT \{x_1(n)x_2(n)\} = \frac{1}{N}[X_1(k) \otimes X_2(k)]$.
- (b) Compute the DFT of the sequence $x(n) = (-1)^n$ for the period $N=16$.
- 6 + 6 = 12**

5. (a) Compute the DFT of the sequence $x(n) = \{1, 0.5\}$ and $h(n) = \{1, 0.5\}$.

(b) What is the difference between discrete-time Fourier transform and continuous-time Fourier transform?

4 + 8 = 12

6. (a) What are the properties that are maintained same in the transfer process of analog filter into a digital filter?
- (b) Prove that physically realizable and stable IIR filters cannot have linear phase.
- (c) An analog filter has the following system function:

$$H(s) = \frac{s + 0.1}{(s + 0.1)^2 + 9}$$
 Convert this analog filter into a digital IIR filter using bilinear transformation method. The digital filter must have a resonant frequency of $\omega_r = \frac{\pi}{4}$.
- 3 + 3 + 6 = 12**

7. (a) Design a lowpass Butterworth IIR digital filter using bilinear transformation technique with $T=1$, from its analog filter that satisfies following specifications:
 Passband cutoff: $\Omega_p = 0.2\pi$; Passband ripple: $R_p = 7dB$
 Stopband cutoff: $\Omega_s = 0.3\pi$; Stopband ripple: $A_s = 16 dB$.
- (b) Transform the analog filter $H_a(s) = \frac{s+1}{s^2+5s+6}$ into a digital filter $H(z)$ using the impulse invariance technique in which $T=0.1$.
- 8 + 4 = 12**

Group - E

8. (a) What are the applications of Multirate signal processing?
- (b) A multirate sampling system is shown in figure below. Determine $y(n)$ as a function of $x(n)$.
9. Let a process $x(n)$ with zero-mean and its autocorrelation is given by $r_x(x) = 10\left(\frac{1}{2}\right)^{|k|} + 3\left(\frac{1}{2}\right)^{|k-1|} + 3\left(\frac{1}{2}\right)^{|k+1|}$
- (i) Find a filter, which when driven by unit variance white noise, will yield a random process with this autocorrelation.
- (ii) Find a stable and causal filter which, when excited by $x(n)$, will produce zero mean, unit variance, white noise.
- 6 + 6 = 12**