OPTIMIZATION TECHNIQUES
(MATH 6121)

Time Allotted: 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) If a function is concave on a convex domain then any local maximum is
(a) global maximum (b) global minimum
(c) local maximum (d) neither global maximum nor global minimum

(ii) Given the optimization problem
$Optimize\ \ f(x,y,z,w)$ subject to $g_1(x,y,z,w) = b_1$ & $g_2(x,y,z,w) = b_2$;
then the order of the bordered Hessian matrix of the Lagrangian function is
(a) 4×4 (b) 5×5 (c) 4×2 (d) 6×6

(iii) The range of λ for which the following payoff matrix is strictly determinable is

PLAYER B

PLAYER A

λ	6	2
-1	λ	-7
-2	4	λ

(a) $\lambda \geq -1$ (b) $\lambda \leq 2$ (c) $-1 \leq \lambda \leq 2$ (d) for any value of λ

(iv) The following is a method of solving a Transportation problem?
(a) North-West Corner method (b) Matrix Minima method
(c) Vogel's approximation method (d) all of (a), (b) & (c)

(v) For the function $f(x) = 3x + 2x^2$, the stationary point is
(a) relative maximum point (b) relative minimum point
(c) global maximum point (d) global minimum point.

(vi) An Assignment problem is
(a) a special class of transportation problem
(b) a linear programming problem
(c) always a maximization type problem
(d) a problem where each job can be done by more than one facility

(vii) If $(-1, 1)$ is a stationary point of the function $f(x, y)$ such that
$$\frac{\partial^2 f}{\partial x^2} = x^2 + y^2, \frac{\partial^2 f}{\partial y^2} = x^2 \text{ and } \frac{\partial^2 f}{\partial x \partial y} = xy$$
Then
(a) $(-1, 1)$ is a local maximum point but not global (b) $(-1, 1)$ is a saddle point
(c) $(-1, 1)$ is a local minimum point but not global (d) $(-1, 1)$ is a local minimum point

(viii) The possible number of basic solutions in a system of 7 equations in 9 unknowns will be
(a) ${}^{9}C_{7}$ (b) ${}^{9}C_{8}$ (c) ${}^{10}C_{9}$ (d) ${}^{8}C_{7}$

(ix) The quadratic form $Q(x, y, z) = x^2 - 2xy + y^2$ is
(a) positive definite (b) positive semi-definite
(c) negative definite (d) indefinite

(x) The Hessian matrix of the function $f(x, y)$ is given by
$$H(f(x,y)) = \begin{pmatrix} -12x^2 - 2 & 2 \\ 2 & -2 \end{pmatrix}.$$
If $(0, 0)$ is a stationary point, then this point would be
(a) a local minimum but not global minimum point
(b) a global maximum point
(c) a saddle point
(d) a local maximum but not global maximum point

Fill in the blanks with the correct word

(xi) The point of inflection occurs at $x = x_0$ for the function $f(x)$ provided $f^{(n)}(x_0) \neq 0$, for n ________.

(xii) The non negative difference between the smallest and second smallest cost in each row and each column in a Transportation problem is called ________.

(xiii) If any variable of the primal problem be unrestricted in sign, then the corresponding dual constraint will be ________.

(xiv) In Big-M method, for a maximization type problem, the coefficient of the artificial variable in the objective function is ________.

(xv) A game is said to have a saddle point if the maximin and minimax values of the game are ________.

Group - B

2. (a) Food X contains 6 units of Vitamin A per gram and 7 units of Vitamin B per gram and costs 12 paise per gram. Food Y contains 8 units of Vitamin A per gram and 12 units of Vitamin B and costs 20 paise per gram. The daily minimum requirements of Vitamins A and B are 100 units and 120 units respectively. Formulate the given problem as an L.P.P. and solve using graphical method to find the minimum cost of the product units. *[(MATH6121.1,MATH6121.2)(Create/HOCQ)]*

(b) Use Simplex method to solve the following linear programming problem:

Maximize $Z = 15x_1 + 6x_2 + 9x_3 + 2x_4$

Subject to

$2x_1 + x_2 + 5x_3 + 6x_4 \leq 20$

$3x_1 + x_2 + 3x_3 + 25x_4 \leq 24$

$7x_1 + x_2 \leq 70$

$x_1, x_2, x_3, x_4 \geq 0.$ *[(MATH6121.1, MATH6121) (Apply/IOCQ)]*

6 + 6 = 12

3. (a) Solve the following linear programming problem using 'Big-M' method:

Maximize $Z = 3x_1 + 2x_2$

Subject to

$2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$ *[(MATH6121.1,MATH6121.2)(Apply/IOCQ)]*

(b) Write the dual form of the given L.P.P.:

Maximize $Z = x_1 + 2x_2$

Subject to

$2x_1 + 4x_2 \leq 160$

$x_1 - x_2 = 30$

$x_1 \geq 10$

$x_1, x_2 \geq 0.$ *[(MATH6121.1, MATH6121.2)(Understand/LOCQ)]*

7 + 5 = 12

Group – C

4. (a) Solve the following transportation problem using VAM, check its optimality and hence find its optimal solution:

	D_1	D_2	D_3	Supply
O_1	4	3	2	10
O_2	1	5	0	13
O_3	3	8	5	12
Demand	8	5	4	

[(MATH6121.1, MATH6121.2, MATH6121.3) (Evaluate/HOCQ)]

(b) Find the minimum cost solution for the 4×4 assignment problem if operator 1 cannot be assigned to machine III and operator 3 cannot be assigned to machine IV: *[(MATH6121.1, MATH6121.2, MATH6121.3) (Apply/IOCQ)]*

	I	II	III	IV
1	5	5	–	2
2	7	4	2	2
3	9	3	5	–
4	7	2	6	7

7 + 5 = 12

5. (a) Find the initial basic feasible solution and the Transportation cost of the given Transportation problem using North-West Corner rule: *[(MATH6121.1, MATH6121.2, MATH6121.3)(Understand/LOCQ)]*

	W_1	W_2	W_3	W_4	Supply
F_1	18	19	49	9	7
F_2	70	30	39	61	9
F_3	42	18	71	25	18
Demand	5	8	7	14	

(b) Find the minimum cost of the given assignment problem whose cost coefficients are given below:

	A	B	C	D
1	2	-1	-1	-2
2	1	0	-2	-1
3	1	-1	-2	0
4	2	2	1	1

[(MATH6121.1MATH6121.2,MATH6121.3)(Apply/HOCQ)]

6 + 6 = 12

Group - D

6. (a) Use graphical method in solving the following game and find the value of the game:

PLAYER B

PLAYER A

2	-1
3	2
-1	5
-2	1

[(MATH6121.1, MATH6121.4) (Understand/LOCQ)]

(b) Use dominance rule to reduce the following pay-off matrix to a 2×2 game and hence find the optimal strategies and the value of the game:

PLAYER B

PLAYER A

3	2	4	0
3	4	2	4
4	2	4	0
0	4	0	8

[(MATH6121.1,MATH6121.4)(Apply/IOCQ)]

6 + 6 = 12

7. (a) Use algebraic method to solve the following game:

PLAYER B

PLAYER A

4	1	2
3	2	1
1	3	4

[(MATH6121.1, MATH6121.4) (Apply/IOCQ)]

(b) In a rectangular game, the pay-off matrix is given by:

PLAYER B

PLAYER A

2	3	2	4	6
0	-2	1	2	1
-1	3	0	-1	3
4	5	-1	2	1
3	2	-2	1	-2

Find the optimal strategies and the value of the game. *[(MATH6121.1, MATH6121.4)(Understand/LOCQ)]*

7 + 5 = 12

Group - E

8. (a) Use Kuhn-Tucker conditions to solve the following non-linear programming problem:

Maximize $Z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$

Subject to the constraints

$x_1 + x_2 \leq 2$

$2x_1 + 3x_2 \leq 12$

$x_1, x_2 \geq 0.$

[(MATH6121.5, MATH6121.6) (Evaluate/HOCQ)]

(b) Find the nature of the function $f(x, y) = 2xy - x^4 - x^2 - y^2$. *[(MATH6121.5, MATH6121.6)(Analyse/IOCQ)]*

8 + 4 = 12

9. Use the method of Lagrange multipliers to solve the following non-linear programming problem. Does the solution maximize or minimize the objective function?

Optimize $Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$

Subject to the constraint

$x_1 + x_2 + x_3 = 15$

$2x_1 - x_2 + 2x_3 = 20$

$x_1, x_2, x_3 \geq 0.$

[(MATH6121.5, MATH6121.6) (Evaluate/HOCQ)]

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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	22.91	36.46	40.63

Course Outcome (CO):

After the completion of the course students will be able to
MATH6121.1 Describe the way of writing mathematical model for real-world optimization problems.
MATH6121.2 Identify Linear Programming Problems and their solution techniques.
MATH6121.3 Categorize Transportation and Assignment problems.
MATH6121.4 Apply the way in which Game Theoretic Models can be useful to a variety of real-world scenarios in economics and in other areas.
MATH6121.5 Convert practical situations into non-linear programming problems.
MATH6121.6 Solve unconstrained and constrained programming problems using analytical techniques.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*