ENGINEERING MATHEMATICS AND BIOSTATISTICS
(BIOT 6121)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
<u>any 4 (four)</u> from Group B to E, taking <u>one</u> from each group.*

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) If λ is an eigenvalue of an orthogonal matrix, which of the following statements is true
(a) $\frac{1}{\lambda}$ is also an eigenvalue
(b) $\frac{1}{\lambda}$ is not an eigenvalue
(c) $\frac{1}{\lambda^2}$ is also an eigenvalue
(d) $\frac{1}{\lambda^3}$ is also an eigenvalue.

(ii) The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + x\left(\frac{dy}{dx}\right)^5 + x = 0$ is
(a) 5 (b) 3 (c) 2 (d) 6.

(iii) Which will be the best statistical test to determine variation in wing length of butterfly from 5 different places?
(a) F-test
(b) Student's t-test
(c) Regression analysis
(d) Chi-square test.

(iv) The rank of a tenth order Identity matrix is
(a) 1 (b) 10 (c) 11 (d) 10^2.

(v) P-value is the probability of the calculated value; if p-value is zero then reject the Ho after comparing with
(a) Level of significance
(b) Degree of freedom
(c) Critical value
(d) Sample size.

(vi) Z-score is calculated for
(a) Chi-square distribution
(b) T-distribution
(c) Standard normal distribution
(d) Normal distribution.

(vii) The degrees of freedom for a Chi Square distribution with 18 samples will be
(a) 17 (b) 18 (c) 19 (d) 20.

(viii) If A and B are two events, then the probability of exactly one of them occurs is given by _____
(a) $P(A \cap B) + P(A \cap B)$
(b) $P(A) + P(B) - 2P(A)\,P(B)$
(c) $P(A) + P(B) - 2P(A)\,P(B)$
(d) $P(A) + P(B) - P(A \cap B)$.

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(ix)   The t-test is based on the t-statistic, which is the ratio of
(a) Sample mean and population mean
(b) Two sample means
(c) Two sample variances
(d) Sample variance and population variance.

(x)   Regression analysis is concerned with
(a) Measuring the extent of association between two variables
(b) Predicting the value of the dependent variable for a given value of the independent variable
(c) Both "Establishing a mathematical relationship between two variables" and "Predicting the value of the dependent variable for a given value of the independent variable"
(d) Establishing a mathematical relationship between two variables.

Fill in the blanks with the correct word

(xi)   A numerical method for solving differential equation is _____.

(xii) When the data are collected from two different groups, it is called a _____ t-test.

(xiii) When the p-value is less than the significance level, the Null hypothesisis _____.

(xiv) If A and B are two events such that P(a) = 0.2, P(b) = 0.6 and P(A /B) = 0.2 then the value of P(A /~B) is _____.

(xv)   In a Binomial Distribution, if 'n' is the number of trials and 'p' is the probability of success, then the mean value is given by _____.

Group - B

2. (a)   Find whether the given system of equations is consistent or not with proper justifications:
$$x + 2y - z = 10$$
$$-x + y + 2z = 2$$
$$2x + y - 3z = 2$$
[(CO)(Analyse/IOCQ)]

(b)   Use Euler's method to approximate y at $x = 1$ given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ and $h = 0.2$.
[(CO1)(Evaluate/HOCQ)]

6 + 6 = 12

3. (a)   Reduce the given matrix, $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ to its row reduced echelon form and hence find its rank.
[(CO1)(Understand/LOCQ)]

(b)   Solve $(1 + x)\frac{dy}{dx} - y = e^{3x}(x + 1)^2$.
[(CO1)(Apply/IOCQ)]

6 + 6 = 12

Group – C

4. (a) In a study on patients the following data was obtained.
Find the median and standard deviation.

Age (in yrs)	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89
No of cases	1	1	1	10	17	38	9	3

[(CO2)(Apply/IOCQ)]

(b) In a certain factory turning out optical lenses there is a small chance of 1/500 for any blade to be defective. The lenses are supplied in a packet of 10. Use poisson distribution to calculate the approximate the number of packets containing no defective, two defective and three defective lenses in a consignment of 10000 packets. (Given $e^{-0.02}=0.9802$). *[(CO2)(Evaluate/IOCQ)]*

6 + 6 = 12

5. (a) In a research laboratory, radioactive emission occurs average one particle per minute. If emission continues for several hundred minutes during which time the particles are emitted randomly, in what proportion of these minutes would we expect the following? (Given $e^{-m}=0.36788$)
(i) Exactly one particle emitted
(ii) Exactly three particles per minute
(iii) More than three particles per minute. *[(CO2)(Evaluate/HOCQ)]*

(b) A couple is heterozygous for albinism (Aa) What is the probability that (i) 4 out of 6 children born to them are normal? (ii) 4 normal and 2 albino out of 6children? *[(CO2)(Deduce/IOCQ)]*

6 + 6 =12

Group - D

6. (a) A sample of 1000 members is found to have a mean of 3.42 cm. Could it be reasonably regarded as a sample from a large population whose mean is 3.30 cm and S.D. is 2.6 cm? Given that $Z_{0.05} = 1.96$. *[(CO3)(Understand/LOCQ)]*

(b) Two samples were drawn from two normal populations. From the following data test whether the two samples have the same variances at 5% level.

Sample-I	66	67	82	75	76	90	92	88	84		
Sample-II	66	64	78	74	87	85	82	95	93	97	92

Critical value of $F_{0.05}$ for df10 and 8 is 3.35. *[(CO3)(Understand/LOCQ)]*

6 + 6 = 12

7. (a) In the evening primrose pure red flowered plants are crossed with white coloured flowered plants. F_1 are all pink coloured. Inbred F_1 plants produced Red 113 Pink 242 White 129. Analyse the result with a suitable statistical test. Given: Critical x^2 value for 2 df is 5.99. *[(CO3)(Analyze/IOCQ)]*

(b) In a sample of owls, it is found that red male 35, red female 70, grey male 50, grey female 45. Colouration is due to the plumage. Is the colouration independent of sex of the sample? Given that Critical value of x^2 at df 1 is 3.84 *[(CO3)(Deduce/IOCQ)]*

6 + 6 = 12

Group – E

8. (a) The following results were obtained from measurement of body length (Y) and body (X) of *Anabus sp*.
    $\Sigma1165$, $\Sigma X^2=56947$, $\Sigma XY=9024.4$ $\Sigma Y=185.2$ $\Sigma Y^2=1437.24$
    Find an appropriate regression line of body length on body weight.   *[(CO4)(Evaluate/HOCQ)]*

   (b) Ten students got the following percentage of marks in mathematics and physics.

Maths(X)	8	36	98	25	75	82	92	62	65	35
Physics(Y)	84	51	91	60	68	62	86	58	35	49

   Find the coefficient of rank correlation.   *[(CO4)(Deduce/IOCQ)]*

   6 + 6 = 12

9. (a) Find the coefficient of correlation between the heights of father and sons from the following data:

Height of fathers (inch)(x)	65	66	67	68	69	70	71
Height of sons (inch)(y)	67	68	66	69	72	72	69

   [(CO4)(Calculate/IOCQ)]

   (b) The following data give the yields on 12 plots of land in three samples under three varieties of fertilizers.

A	B	C
25	20	14
22	17	26
24	16	30
21	19	20

   Is there any significant difference in the average yields of land under the three varieties of fertilizers? Given that *F* at *df* (2, 9) at 5% level = 4.26

   [(CO4)(Calculate/HOCQ)]

   6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	18.75	56.25	25

Course Outcome (CO):

After completing the course, the students will be able to:
1. Understand the basic principles of linear algebra, concept of function, limit, continuity and normal distribution.
2. Comprehend and use the different statistical models of dispersion and probability dispersion.
3. Apply the mathematical and biostatistical models in biological systems for testing of hypotheses, estimation of group differences and case-control studies.
4. Interpret the concept of correlation and regression analysis of variables along with analysis of variance.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.