FINITE ELEMENT METHOD
(MECH 3231)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The coefficient in stress-strain relation for a linear, elastic, isotropic material under plane strain condition is given by

(a) $\frac{E}{(1-2\nu)}\begin{bmatrix} 1-\nu & 0 & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$ (b) $\frac{E}{(1+\nu)}\begin{bmatrix} 1-\nu & 0 & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$

(c) $\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} \nu & 0 & 0 \\ \nu & \nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$ (d) $\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$

(ii) [B] matrix is the coefficient matrix of displacement matrix in
(a) Stress-Strain relation (b) Stain-Displacement relation
(c) Force-Displacement relation (d) Stress-Displacement relation.

(iii) Which of the following method is based on PSTP (Principle of Stationary Total Potential)?
(a) Galarkin Weighted Residual (b) Domain Residual Method
(c) Raileigh-Ritz Method (d) All of the above.

(iv) Expression of deflection in terms of shape function (N_1, N_2, N_3 & N_4) for a BEAM element is
(a) $v = N_1v_1 + N_2\theta_1 + N_3v_2 + N_4\theta_2$ (b) $v = N_1v_1 + N_2v_2 + N_3v_3 + N_4v_4$
(c) $v = N_1\theta_1 + N_2\theta_2 + N_3\theta_3 + N_4\theta_4$ (d) $v = N_1 + N_2 + N_3 + N_4$

(v) Elemental stiffness matrix of a BAR element transformed to global coordinate of a plane Truss is a
(a) [3 × 3] matrix (b) [4 × 4] matrix
(c) [2 × 2] matrix (d) [6 × 6] matrix.

(vi) Elemental degree of freedom for a Frame element is
(a) 4 (b) 2 (c) 6 (d) 1.

(vii) For a Four-Noded Rectangular element if one of the shape functions in user-coordinate (x, y) is expressed as $N_1 = \left(1 - \frac{x}{l}\right)\left(1 - \frac{y}{w}\right)$, then the corresponding shape function in normalized coordinate (ξ, η) will be?
(a) $\frac{1}{4}(1-\xi)(1+\eta)$ (b) $\frac{1}{4}(1-\xi)(1-\eta)$ (c) $\frac{1}{2}(1-\xi)(1-\eta)$ (d) $\frac{1}{4}(1-\xi)$

(viii) CST element possesses
(a) Constant field variable throughout the element
(b) Derivative of the field variable is constant throughout the element
(c) Variation of the field variable is quadratic throughout the element
(d) Variation of the field variable is cubic throughout the element.

(ix) Iso-parametric elements are those who use same shape function for both
(a) stress calculation and strain calculation
(b) interpolation of field variable and geometry transformation
(c) displacement and stress calculation
(d) displacement and strain calculation.

(x) The sequence of the numerical simulation in any FEA software is
(a) Pre-processing→Solution→Post-processing
(b) Post-processing→Solution→Pre-processing
(b) Pre-processing→Post-processing→Solution
(d) Solution→ Pre-processing→Post-processing.

Fill in the blanks with the correct word

(xi) If a BAR element has length 1000mm, cross-section 10mm × 10mm and modulus of elasticity is 210 GPa them its stiffness is _____ N/mm.

(xii) An beam imposed with transversed as well axial load should be analysed with FEM using a _____ Element.

(xiii) If an object (without branches) in a problem being solved with FEM is discretized with 5 numbers of plane FRAME element, then its global stiffness matrix would be a _____×_____ matrix.

(xiv) In solving a problem related to stretching of a thin disc, we should adopt plain _____ condition as the stress-strain constitutive relation.

(xv) For the structural analysis of a gear tooth using FEM by representing the tooth topology in 3-dimension, the most suitable element for meshing is _____.

Group - B

2. For the following differential equation and stated boundary conditions, obtain a two-term solution using Galerkin's method of weighted residuals using the specified trial functions. Compare the two-term solution to the exact solution. Also find the value of 'y' at x=0.4 by the both solution methods and compare their values.

$\frac{d^2y}{dx^2} - 5x^2 + 7 = 0, \qquad 0 \leq x \leq 1$

The given boundary conditions are: $y(0) = 0$ and $y(1) = 0$

The trial function is: $N_1(x) = x(1-x)$ and $N_2(x) = x^2(1-x)$ *[(CO1)(Analysis/IOCQ)]*

(8 + 3 + 1) = 12

3. Two trolleys are connected by the arrangement of springs shown in Fig. 1 below. (i) Determine global stiffness matrix using Rayleigh-Ritz method (ii) If k=10 N/mm, F_1 = 100N and F_2 = 70N, compute the displacement of each trolley and the force in each spring. *[(CO2)(Analyse/HOCQ)]*

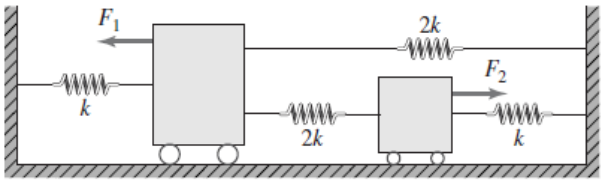

Fig. 1

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Group - C

4. (a) Schematically represent properly a BAR element. Write its interpolation function in terms Shape Function and nodal values of deflections. Write also the expressions of Shape functions for a BAR element in terms of domain variable 'x' and element length 'l'. *[(CO2)(Remember/LOCQ)]*

(b) Fig. 2 depicts an assembly of two bar elements made of two different materials. Determine global stiffness matrix of this problem. Write down the FEA equation to determine displacements at junction of two different cross-sections and at free end of the Bar. Write also the FEA equations to determine stress in the bar and reaction force at fixed end.

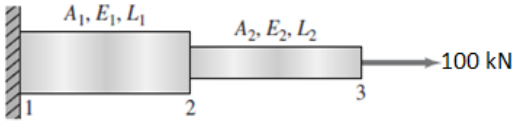

Fig. 2

Given A_1 = 2500 mm², A_2 = 1500 mm², E_1 = 210 GPa, E_2 = 257 GPa and $L_1 = L_2$ = 5m

[(CO2)(Apply/IOCQ)]

(1 + 1 + 2) + (4 + 2 + 2) = 12

5. Consider the beam shown in Fig. 3 below. The beam has been loaded as shown in figure and is made of a material having modulus of elasticity E = 10×10^6 psi. Now find global stiffness matrix for this problem and write down the FEA equations. Use 3 beam elements, assemble and incorporate boundary conditions. Solution not required.

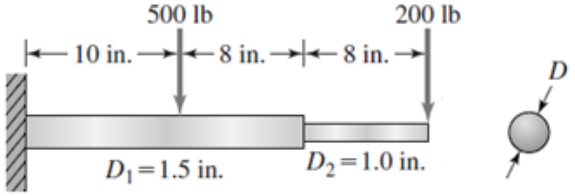

Fig. 3

[(CO2)(Apply/HOCQ)]

(4 + 6 + 2) = 12

Group - D

6. (a) Briefly discuss about plane strain condition with a suitable example. Also write down the stress-strain constitutive relation for a linear, elastic, isotropic material under this plane strain condition. *[(CO4)(Sketch/LOCQ)]*

(b) Derive the expression of shape functions of triangular element known as TRIA in terms of Cartesian coordinates x and y. *[(CO4)(Apply/IOCQ)]*

(3 + 2) + 7 = 12

7. (a) What do you understand by iso-parametric element? Write a short note on 'Normalized Co-ordinate' system. *[(CO5)(Sketch/LOCQ)]*

(b) Schematically represent a quadrilateral element in normalized coordinate system and write down expression of shape functions of the quadrilateral element in normalized coordinate system. *[(CO5)(Apply/LOCQ)]*

(2 + 3) + (3 + 4) = 12

Group - E

8. (a) Fig. 4 shows a quadrilateral element in global coordinates. Show that the mapping correctly describes the line connecting nodes 2 and 3 and determine the (x, y) coordinates corresponding to $(\xi, \eta,)$= (0.4, 0.7). *[(CO5)(Analyse/IOCQ)]*

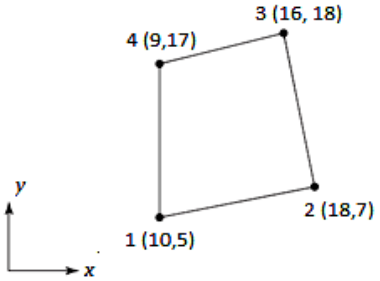

Fig. 4

(b) Evaluate the given integral analytically and using Gauss–Legendre formula.
$\int_1^5 (x^3 + 7x^2 + 12)\, dx$ *[(CO5)(Analyse/IOCQ)]*

6 + 6 = 12

9. Describe in detail about classification of elements based on topology used in a FEA software. In your description represent each element schematically showing all the nodal degree of freedoms. *[(CO6)(Remember/IOCQ)]*

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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	21.88	53.12	25

Course Outcome (CO):

After the completion of the course students will be able to

CO1 Explain the transformation of numerical approach to numerical approach to solve a structural problem.

CO2 Illustrate the expressions of Shape Functions of different 1D elements like BAR, BEAM and FRAME used to solve a structural problem

CO3 Apply 1D elements like BAR element, BEAM element and FRAME element correctly to solve a structural problem numerically using the FEA method.

CO4 Distinguish the 'Plane Stress' approach and 'Plane Strain' approach to solve any structural problem numerically using the FEA method with 2-Dimensional elements like 'TRIA' and 'QUAD'.

CO5 Choose 'Normalized Coordinate System' in place of 'User Coordinate System' in solving a structural problem numerically using the FEA method.

CO6 Adapt proper steps of operation of any FEA software like MSC Software, ANSYS etc using computer as working or solving media.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*