COMPUTATIONAL FLUID DYNAMICS
(MECH 3221)

Time Allotted: 2½ hrs **Full Marks: 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) Unsteady flow state means
(a) Flow parameters are non-uniform in space
(b) Flow parameters are uniform in space
(c) Flow parameters are time-dependent
(d) Flow parameters are time-independent.

(ii) Consider the following statements:
I. Pressure force is a type of surface force
II. Pressure force is a type of body force
III. Gravity is a type of body force
IV. Coriolis force is a type of body force of electromagnetic origin
Which of the following statement(s) is/are incorrect?
(a) Only (I) and (III) (b) Only (II) and (III)
(c) Only (II) and (IV) (d) (I), (III) and (IV).

(iii) In the context of a second-order PDE for a 2D problem, a characteristic is defined as
(a) A line across which the first order derivatives are discontinuous
(b) A line across which the second order derivatives are discontinuous
(c) A surface across which the first order derivatives are discontinuous
(d) A surface across which the second order derivatives are discontinuous

(iv) The various terms included in the generalized transport equation are
(a) Rate of change term, radiative term, convective term, source term
(b) Rate of change term, convective term, diffusive term, source term
(c) Advective term, diffusive term, convective term, source term
(d) Rate of change term, diffusive term, convective term, advective term

(v) Choose the correct statement on the nature of partial differential equations:
(a) If the equation has no real characteristic, the equation is elliptic
(b) If the equation has one real characteristic, the equation is elliptic
(c) If the equation has one real characteristic, the equation is hyperbolic
(d) If the equation has two real and distinct characteristics, the equation is parabolic.

(vi) Consider the following statements:

I. The tri-diagonal matrix algorithm (TDMA) is a technique for rapidly solving tri-diagonal systems.
II. The TDMA is actually an iterative method for one-dimensional problems.
III. The TDMA can be applied iteratively, in a line-by-line fashion, to solve multi-dimensional problems.
IV. The TDMA is computationally expensive and requires a large amount of storage.

Which of the following statement(s) is/are incorrect?

(a) Only (I) (b) (II) and (IV) (c) (II) and (III) (d) Only (IV)

(vii) Consider 1D steady state conduction without volumetric heat generation through a rod of length L. The heat flux at x= 0 and at x= L are each equal to q W/m^2. Following statements are given pertaining to the numerical solution of the above problem:

I. Unique temperature profile cannot be obtained.
II. The problem is ill-posed.
III. Tri-diagonal matrix algorithm (TDMA) works to solve the problem

Which of the following statement (s) is/are correct?

(a) (I), (II) and (III) (b) (I) and (II) only (c) (I) only (d) (II) only

(viii) Which of the following statements is incorrect in case of finite volume discretization?

(a) If the source term is linearized as $S = S_C + S_P T_P$ then S_P may be of any sign
(b) If the source term is linearized as $S = S_C + S_P T_P$ then S_P should be negative
(c) If the source term is linearized as $S = S_C + S_P T_P$ then S_C should be negative
(d) If the source term is linearized as $S = S_C + S_P T_P$ then S_C should be positive.

(ix) A staggered grid system is used mainly to

(a) Check the propagation of round-off error
(b) Enable treatment of flow domain of irregular shapes
(c) Check the propagation of truncation error
(d) Eliminate possibilities of highly irregular checker-board pressure field

(x) The complexity and size of the set of linear algebraic equations depends on

(a) Dimensionality of the problem (b) Number of grid nodes
(c) Discretisation practice (d) All of the above.

Fill in the blanks with the correct word

(xi) The number of operations to the solution of a system of N equations with N unknowns by means of a direct method is of the order of _____.

(xii) Equation discretization is the process of converting _____ PDEs into discretized equations.

(xiii) Discretization of the governing equations result in _____.

(xiv) The minimum number of vertices that a 3-D element can have is _____.

(xv) Cramer's rule matrix inversion and Gaussian elimination are examples of _____ of solution techniques for linear algebraic equations.

Group - B

2. (a) What is the full form of CFD? State two principal reasons behind the upsurge of interest in CFD within industrial community. State any three unique advantages of CFD over experiment-based approaches. *[(CO1)(Understand/LOCQ)]*

   (b) Mention the primary user activities at the pre-processing stage. Why are optimal meshes non-uniform? *[(CO6)(Understand/LOCQ)]*

(1 + 2 + 3) + (5 + 1) = 12

3. (a) Calculate the value of substantial derivative $\left(\frac{D\vec{V}}{Dt}\right)_{t=2s}$ of the velocity field represented by $\vec{V} = (-2xt)\hat{i} + (-3y)\hat{j} + (4zt^2)\hat{k}\left(\frac{m}{s}\right)$ at a point P (1, -3, 4). Write down the values of the temporal and convective components of acceleration. *[(CO2)(Apply/IOCQ)]*

   (b) In Fig. 1 below, three temperature distribution profiles (**A**, **B**, or **C**) are shown for a conducting rod, without any heat generation source, maintained at 0°C at one end and 100°C at the other. Which of these profiles represents a physically consistent solution? Why? *[(CO2)(Analyze/IOCQ)]*

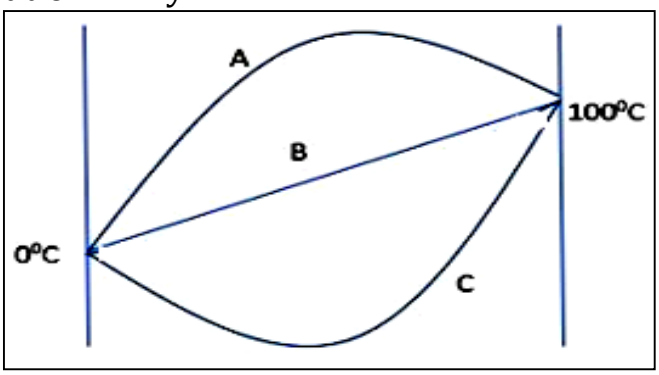

Fig. 1

(4 + 2 + 2) + (2 + 2) = 12

Group – C

4. (a) Consider the source term in the form $S = 3 + 4T$. Consider the following recipes for source term linearization $(S = S_C + S_P T_P)$:

   i. $S_C = 3;\ \ S_P = 4$

   ii. $S_C = 3 + 4T_P^*;\ \ S_P = 0$

   iii. $S_C = 3 + 8T_P^*;\ \ S_P = -4$

   Which of the following source term linearization is/are correct? Why? *[(CO3)(Analyse/IOCQ)]*

   (b) Write a short note on (i) transportiveness and (ii) boundedness of a discretization scheme for handling convection-diffusion coupled problems. *[(CO4)(Understand/LOCQ)]*

(1 + 3) + (4 + 4) = 12

5. (a) Consider 1D, steady-state, source-free heat conduction in an insulated metallic rod of 0.6 m length, whose ends are maintained at constant temperature of 50°C and 250°C respectively. The thermal conductivity and cross sectional area of the rod are $k = 1000\ \frac{W}{mK}$ and A = 0.015m² respectively. Find out the set of algebraic equations for the temperature distribution along the rod using finite volume

method, by dividing the rod into four equal control volumes. Represent the algebraic equations in the matrix form. *[(CO3)(Apply/IOCQ)]*

(b) Write the general transport equation for a general scalar variable Ø per unit mass, mentioning the physical meaning of the respective terms. *[(CO2)(Understand/LOCQ)]*

8 + (1 + 3) = 12

Group - D

6. (a) Describe the 'SIMPLER' algorithm in flowchart form. *[(CO5)(Apply/IOCQ)]*

(b) The 'SIMPLER' algorithm an improvement over the SIMPLE' algorithm: Justify. *[(CO5)(Apply/IOCQ)]*

8 + 4 = 12

7. Solve the following matrix equation using the TDMA:

$$\begin{bmatrix} 350 & -120 & 0 & 0 & 0 \\ -120 & 250 & -120 & 0 & 0 \\ 0 & -120 & 250 & -120 & 0 \\ 0 & 0 & -120 & 250 & -120 \\ 0 & 0 & 0 & -120 & 350 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 28000 \\ 5000 \\ 5000 \\ 5000 \\ 56000 \end{bmatrix}$$

[(CO5)(Apply/IOCQ)]

12

Group - E

8. (a) Consider a rectangular pipe with uniform cross-section and prescribed values of length and cross-sectional dimensions. The ambient conditions of temperature and pressure are specified. Water is flowing through the pipe under an imposed pressure head. The bottom wall is maintained at a constant value of high temperature (specified) throughout the length of the pipe. Regard all other walls as adiabatic. Outline the procedural steps to solve this problem using a Computational Fluid Dynamics (CFD) software that you are familiar with, assuming laminar flow regimes throughout the flow field. *[(CO6)(Evaluate/HOCQ)]*

(b) For the above problem, write down the steps needed to obtain the wall averaged Nusselt number and the wall-averaged friction factor. *[(CO6)(Evaluate/HOCQ)]*

8 + 4 = 12

9. Write short notes on (i) Boundary conditions (ii) Pre-processor (iii) Structured grid. *[(CO6)(Apply/IOCQ)]*

(4 + 4 + 4) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	25	62.50	12.50

Course Outcome (CO):

After the completion of the course students will be able to

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CO1: Describe the fundamental conservation laws of fluid mechanics.

CO2: Express the transport equations in general form.

CO3: Construct the methodologies for converting Partial Differential Equations (PDE) to discretised algebraic forms using Finite Volume Method (FVM).

CO4: Analyze various CFD solution algorithms for steady and unsteady flows.

CO5: Compare the results obtained from direct analytical solution and FVM using Tri-Diagonal Matrix Algorithm (TDMA).

CO6: Formulate CFD problems using CFD software and examine the validity of such schemes.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*