ADVANCED NUMERICAL METHODS
(MATH 2202)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The value of $\|A\|_1$, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is

(a) 24 (b) 18 (c) 15 (d) 12.

(ii) To generate the j^{th} experiment in Golden Section Search algorithm, the length of the j^{th} experiment L_j is given by

(a) $\frac{1}{\gamma^j} L_0.$ (b) $\frac{1}{\gamma^{j+1}} L_0.$ (c) $\frac{1}{\gamma^{j-1}} L_0.$ (d) $\frac{1}{\gamma^{2j}} L_0.$

L_0 being the initial interval of uncertainty and γ is the golden ratio.

(iii) Choose the correct statement:

(a) $\|\alpha A\| \neq |\alpha|\|A\|$, ($\alpha$ is a constant) (b) $\|A\| = 0 \ iff \ A < 0$

(c) $\|A + B\| > \|A\| + \|B\|$ (d) $\|A + B\| \geq \|AB\|$.

(iv) The value of $\Delta^3[(1 - x)(1 - 2x)(1 - 3x)]$ taking $h = 1$ is

(a) 36 (b) 37 (c) -35 (d) -36.

(v) In the QR decomposition of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$, the matrix Q is

(a) $\begin{bmatrix} 1 & \sqrt{2} \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ (c) $\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix}$.

(vi) For the system $AX = B$ in Cholesky's factorization method, the matrix A has to be

(a) anti-symmetric (b) singular

(c) symmetric positive definite (d) negative definite.

(vii) $[x_0, x_1, x_2, x_3]$ equals

(a) $\frac{[x_1, x_2, x_3]-[x_0,x_1,x_2]}{x_3-x_0}$ (b) $\frac{[x_1, x_2, x_3]-[x_0,x_2,x_3]}{x_2-x_1}$

(c) $\frac{[x_2, x_3]-[x_0,x_1]}{x_3-x_0}$ (d) $\frac{[x_1, x_2, x_3]-[x_0,x_1,x_2]}{x_0-x_3}$.

(viii) If $f(x)$ is a cubic spline in the interval (x_0, x_n) then
(a) $f(x)$ is linear polynomial outside the interval (x_0, x_n)
(b) $f(x)$ is quadratic polynomial outside the interval (x_0, x_n)
(c) $f(x)$ is cubic polynomial outside the interval (x_0, x_n)
(d) $f(x)$ is trigonometric function outside the interval (x_0, x_n).

(ix) The interval containing all the eigen values of the symmetric matrix $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ is

(a) $[-1, 7]$ (b) $[-1, 9]$ (c) $[-7, 7]$ (d) $[1, 9]$.

(x) The spectral radius of the matrix $\begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{4} \\ -\frac{1}{3} & 0 & \frac{1}{2} \\ -\frac{1}{4} & -\frac{1}{2} & 0 \end{bmatrix}$ is

(a) < 0 (b) < 1 (c) $< \frac{7}{12}$ (d) $< \frac{3}{4}$.

Fill in the blanks with the correct word

(xi) In Internal halving method, the initial interval of uncertainty is divided into ___________ equal parts.

(xii) The total number of multiplications or divisions in Gauss-elimination method with n equations (large n) is _________.

(xiii) The singular values of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}$ are _________.

(xiv) A search algorithm which is more efficient than Dichotomous Search algorithm is ____________.

(xv) Geometrically, Simpson's one third rule for three points of interpolation represents a ____________.

Group - B

2. (a) (i) Is the following matrix positive definite? Justify your answer. $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

(ii) Find the value of infinity norm and Euclidean norm of the matrix $A = \begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$

[(MATH2202.1, MATH2202.4, MATH2202.6)(Remember/LOCQ)]

(b) Find the solutions of the given system of equations by Gauss-Jacobi's method, correct to 2 significant figures.

$$4x + 11y - z = 33$$
$$x + y + 4z = 9$$
$$8x - 3y + 2z = 20.$$

[(MATH2202.1,MATH2202.4,MATH2202.6)(Apply/IOCQ)]

(2 + 4) + 6 = 12

3. (a) What do you mean by partial pivoting?
What is its basic difference from complete pivoting?
Now use Gauss elimination method with partial pivoting to solve the following system of equations:

$x_1 - 2x_2 + 3x_3 = 9$
$2x_1 + x_2 - x_3 = -1$
$3x_1 - x_2 + 5x_3 = 14.$

[(MATH2202.1, MATH2202.4, MATH2202.6)(Understand/LOCQ)]

(b) Find the value of $\| A \|_1$ and $\| A \|_e$ for the following matrix $A = \begin{bmatrix} 10 & -7 & 0 \\ -3 & 2.099 & 6 \\ 5 & -1 & 5 \end{bmatrix}$.

[(MATH2202.1, MATH2202.4, MATH2202.6)(Remember/LOCQ)]

(2 + 1 + 6) + 3 = 12

Group - C

4. (a) Sketch the Gerschgorin's circles to locate the eigenvalues of the matrix

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{2} & 0 & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \end{bmatrix}.$$

Shade the smallest region containing all the eigenvalues of A.

[(MATH2202.3, MATH2202.4, MATH2202.6)(Understand/LOCQ)]

(b) Find the least eigenvalue and the corresponding eigenvector for the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ after four iterations of the inverse iteration method using the initial approximation as $X_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

[(MATH2202.3, MATH2202.4, MATH2202.6)(Apply/IOCQ)]

5 + 7 = 12

5. Find the singular values, and hence the Singular Value Decomposition of the matrix $\begin{bmatrix} 3 & -1 \\ 1 & 3 \\ 1 & 1 \end{bmatrix}$.

[(MATH2202.3, MATH2202.4, MATH2202.6)(Evaluate/HOCQ)]

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Group - D

6. (a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using:

(i) Trapezoidal rule, (ii) Simpson's $\frac{3}{8}^{th}$ rule, taking $n = 6$.

[(MATH2202.2, MATH2202.6)(Understand/LOCQ)]

(b) From the following table, estimate the number of students who obtained marks between 40 and 45.

Marks:	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students:	31	42	51	35	31

[(MATH2202.2, MATH2202.6)(Analyze/IOCQ)]

(3 + 3) + 6 = 12

7\. (a) Using Lagrange's interpolation formula, express $\frac{3x^2+x+1}{x^3-6x^2+11x-6}$ as the sum of partial fractions. *[(MATH2202.2, MATH2202.6)(Understand/LOCQ)]*

(b) Find the cubic spline corresponding to the interval $[1, 2]$ for the following values:

x	1	2	3
y	-6	-1	16

Hence evaluate $f(1.5)$. *[(MATH2202.2, MATH2202.6)(Evaluate/HOCQ)]*

6 + 6 = 12

Group - E

8\. Find the value of x in the interval $[0, 1]$ which minimizes the function $f(x) = x^2(x - 2.5)$ using Fibonacci Search algorithm using 6 functional evaluations.

[(MATH2202.5, MATH2202.6)(Apply/IOCQ)]

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9\. Minimize the function $f(x) = 2 - 4x + e^x$ in the interval $[0.5, 2.5]$ with an accuracy of $\epsilon = 0.0002$ using Dichotomous Search. Take the tolerance value to be 0.3. Also, find the reduction ratio. *[(MATH2202.5,MATH2202.6)(Apply/IOCQ)]*

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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	36.46	44.79	18.75

Course Outcome (CO):

After the completion of the course students will be able to

MATH2202.1 Analyze certain algorithms, numerical techniques and iterative methods that are used for solving system of linear equations.

MATH2202.2 Implement appropriate numerical methods for solving advanced engineering problems dealing with interpolation, integration and differentiation.

MATH2202.3 Apply the knowledge of matrices for calculating eigenvalues and eigenvectors and their stability for reducing problems involving Science and Engineering

MATH2202.4 Develop an understanding to reduce a matrix to its constituent parts in order to make certain subsequent calculations simpler.

MATH2202.5 Apply various optimization methods for solving realistic engineering problems.

MATH2202.6 Compare the accuracy and efficiency of the above mentioned methods.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*