OPERATIONS RESEARCH
(MATH 2203)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) Which of the following Hessian matrices is positive definite?
(a) $\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}$.

(ii) If the primal and dual do not have a feasible solution, then
(a) dual objective function is unbounded
(b) finite optimal for both exists
(c) primal objective function is unbounded
(d) finite optimal for both do not exist.

(iii) The value of the following game problem is

PLAYER B

PLAYER A	B_1	B_2	B_3
A_1	1	3	1
A_2	0	−4	−3
A_3	1	5	−1

(a) −4 (b) 1 (c) 3 (d) 0.

(iv) The bordered Hessian matrix of the Lagrangian function L constructed for an optimization problem with equality constraints is given by

$$H^B(L) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 3 \\ 1 & 2 & 2 & 0 \\ 1 & 3 & 0 & 2 \end{pmatrix}$$

then the number equality constraints involved in the problem is,
(a) 1 (b) 2 (c) 3 (d) 0.

(v) For the standard minimization problem, the Kuhn-Tucker necessary conditions are also sufficient conditions, provided
(a) objective function is convex and constraints are concave
(b) objective function and constraints are concave
(c) objective function and constraints are convex
(d) objective function is concave and constraints are convex.

(vi) In a fair game, the value of the game is
(a) 1 (b) 0 (c) unbounded (d) 2 .

(vii) To convert ≥ inequality into equality constraints in a L.P.P, we must
(a) subtract an artificial variable
(b) add a surplus variable
(c) subtract a surplus variable
(d) add a slack variable.

(viii) The degeneracy in the transportation problem indicates that
(a) the problem has no feasible solution
(b) more than one optimal solution exists
(c) dummy allocation(s) are needed
(d) the solution is non-degenerate.

(ix) The total possible number of ways of assigning 6 jobs to 6 persons is
(a) 6^2 (b) ${}^6C_6$ (c) 6! (d) 6^6 .

(x) For a maximization type LP problem, the coefficient of the objective function for an artificial variable is
(a) $-M$ (b) M (c) 0 (d) ∞.

Fill in the blanks with the correct word

(xi) If a function is concave on a convex domain then any local maximum is _________.

(xii) In a two person zero sum game gains of one player are equal to the ________ of the other player.

(xiii) While solving an L.P.P using graphical method, the area bounded by the constraints in the positive quadrant is called the ___________________.

(xiv) When total supply becomes equal to the total demand, the transportation problem is said to be _________.

(xv) In Fibonacci Search algorithm, if 6 iterations are performed, the reduction ratio is given by ___________.

Group - B

2. (a) Find the graphical solution of the given L.P.P :

Maximize $z = 2x_1 + 3x_2$
subject to the constraints

$$2x_1 + 4x_2 \leq 4$$
$$2x_1 + 3x_2 \geq 6$$
$$x_1,\ x_2 \geq 0.$$

[(MATH2203.1, MATH2203.2) (Understand/LOCQ)]

(b) Solve the following L.P.P by simplex algorithm:

Maximize $z = 4x_1 + 3x_2$
subject to the constraints

$$2x_1 + x_2 \leq 30$$
$$x_1 + 2x_2 \leq 24$$
$$x_1,\ x_2 \geq 0.$$

[(MATH2203.1, MATH2203.2) (Apply/IOCQ)]

5 + 7 = 12

3. (a) Solve the following L.P.P by Big-M method:

Maximize $z = x_1 + x_2$
subject to the constraints

$$x_1 + x_2 \leq 2$$
$$-3x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0.$$

[(MATH2203.1, MATH2203.2) (Apply/IOCQ)]

(b) Find the dual of the given primal

Minimize $z = 2x_1 - 4x_2 + 5x_3$
subject to the constraints

$$3x_1 + 4x_2 + x_3 \geq 2$$
$$x_1 - 5x_3 \geq 3$$
$$x_1 - 3x_2 + x_3 = -4$$

$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

[(MATH2203.1, MATH2203.2) (Understand/LOCQ)]

7 + 5 = 12

Group - C

4. (a) Solve the given transportation problem using VAM and hence find it's optimal solution:

	D_1	D_2	D_3	D_4	Availability
S_1	3	8	7	4	30
S_2	5	2	9	5	50
S_3	4	3	6	2	80
Demand	20	60	55	40	

[(MATH2203.1, MATH2203.2 MATH2203.3, MATH2203.4) (Evaluate/HOCQ)]

(b) Solve the assignment problem where the cost of assigning operators **I to V** to Machines **A to E** is given below:

	A	**B**	**C**	**D**	**E**
I	10	5	13	15	16
II	3	9	18	13	6
III	10	7	2	2	2
IV	7	11	9	7	12
V	7	9	10	4	12

[(MATH2203.1, MATH2203.2 MATH2203.3, MATH2203.4) (Evaluate/HOCQ)]

7 + 5 = 12

5. (a) Reduce the following game to a 2×2 game graphically and hence solve it.

PLAYER B

PLAYER A	2	−1	5	−2	6
	−2	4	−3	1	0

[(MATH2203.1, MATH2203.2 MATH2203.3, MATH2203.4) (Apply/IOCQ)]

(b) Use the Algebraic method to solve the following game:

PLAYER B

	4	1	2
PLAYER A	3	2	1
	1	3	4

[(MATH2203.1, MATH2203.2 MATH2203.3, MATH2203.4) (Understand/LOCQ)]

6 + 6 = 12

Group - D

6. (a) Use Kuhn-Tucker conditions to solve the following non-linear programming problem:

Maximize $z = 2x_1 - x_1^2 + x_2$
subject to the constraints

$$2x_1 + x_2 \leq 4$$
$$2x_1 + 3x_2 \leq 6$$
$$x_1, x_2 \geq 0.$$

[(MATH2203.6) (Evaluate/HOCQ)]

(b) Find the nature of the following function:

$$f(x_1, x_2, x_3) = x_1^4 + (x_1 + x_2)^2 + (x_1 + x_3)^2.$$

[(MATH2203.6) (Understand/LOCQ)]

8 + 4 = 12

7. Use the method of Lagrangian multipliers to solve the following non- linear programming problem. Does the solution maximize or minimize the objective function?

Optimize $z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3$
subject to the constraint

$$x_1 + x_2 + x_3 = 7$$
$$x_1, x_2, x_3 \geq 0$$

[(MATH2203.6) (Evaluate/HOCQ)]

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Group - E

8. Find the minimum of $f(x) = x^3 - 42x^2 - 31x + 12$ in the interval [0.0, 2.0] using interval halving method taking the tolerance to be less than 0.3. *[(MATH2203.1, MATH2203.5) (Apply/IOCQ)]*

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9 Use Fibonacci search algorithm to minimize $f(x) = x^3 - 13x^2 + 11x - 3$ in $[0,\ 2]$ using 6 functional evaluations.

[(MATH2203.1, MATH2203.5) (Apply/IOCQ)]

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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	33.3	33.3	33.4

Course Outcome (CO):

After the completion of the course students will be able to

MATH2203.1 Describe the way of writing mathematical model for real-world optimization problems.
MATH2203.2 Identify Linear Programming Problems and their solution techniques.
MATH2203.3 Categorize Transportation and Assignment problems.
MATH2203.4 Apply the way in which Game theoretic models can be useful to a variety of real-world scenarios in economics and in other areas.
MATH2203.5 Apply various optimization methods for solving realistic engineering problems and compare their accuracy and efficiency.
MATH2203.6 Convert practical situations into non-linear programming problems and solveunconstrained and constrained programming problems using analytical techniques.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*