MATHEMATICS - II
(MTH 1201)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) If for a random variable X, $Var(X) = 1$, then $Var(5X - 2)$ is
(a) 25 (b) 23 (c) 5 (d) 2

(ii) One of the roots of $x^3 - 17x + 5 = 0$ lies in between
(a) 1 and 2 (b) 0 and 1 (c) 2 and 3 (d) -1 and 0

(iii) If A and B are independent events then $P(A^c/B^c)$ is
(a) $P(A^c)$ (b) $P(A^c \cap B^c)$ (c) $P(B^c)$ (d) $P(A^c \cup B^c)$

(iv) Runs scored by a batsman in 5 one day matches are 50, 70, 82, 93, and 20. Then the standard deviation of the score is
(a) 7.9 (b) 25.8 (c) 63 (d) 665.6

(v) System of linear equations may be solved by
(a) Newton-Raphson method (b) Gauss-Seidel method
(c) Euler's method (d) Runge-Kutta method

(vi) A binary tree should have at least
(a) one vertex (b) two vertices.
(c) three vertices (d) four vertices

(vii) The value of $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ is
(a) $\frac{\sqrt{3}\pi}{2}$ (b) $\frac{2\pi}{\sqrt{3}}$ (c) $\frac{2\pi}{\sqrt{3}}$ (d) $\frac{\pi}{2}$

(viii) $\mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s+2)}\right\}$ is equal to
(a) $\cos at$ (b) $cosh\, at$ (c) $sin\, at$ (d) $sinh\, at$

(ix) The value of $\int_0^{\frac{\pi}{2}} sin^3x\, cos^{\frac{5}{2}}x\, dx$ is
(a) $\frac{88}{7}$ (b) $\frac{8}{11}$ (c) $\frac{88}{77}$ (d) $\frac{8}{77}$.

(x) The maximum degree of any vertex in a simple graph with 10 vertices is
(a) 10 (b) 5 (c) 20 (d) 9

Fill in the blanks with the correct word

(xi) Rounding off the number 0.004935 to 3 significant figures, we get _____.

(xii) If A and B are events with $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cap B) = \frac{1}{4}$ then $P(A^c \cap B) =$ _____.

(xiii) The convolution of $f(t) = t$ and $g(t) = 1$ is _____.

(xiv) A binary tree has exactly one vertex of degree _____.

(xv) The probability that a leap year selected at random will contain 53 Sundays is _____.

Group - B

2. (a) The probability density function of a random variable X is
$f(x) = k(x-1)(2-x)$ for $1 \leq x \leq 2$.
Determine
(i) the value of k,
(ii) $P\left(\frac{5}{4} \leq X \leq \frac{3}{2}\right)$. *[(MTH1201.1,MTH1201.2)(Understand /LOCQ)]*

(b) The chance that a doctor will diagnose a certain disease correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of the doctor, who had the disease, dies. What is the probability that the disease was diagnosed correctly? *[(MTH1201.1, MTH1201.2)(Understand/LOCQ)]*

6 + 6 = 12

3. (a) Marks obtained by 1000 students to a final examination are found to be normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be
(i) between 60 and 75, both inclusive,
(ii) below 40. *[(MTH1201.1,MTH1201.2)(Evaluate/HOCQ)]*

(b) Six dice are thrown 729 times. How many times do you expect at least three dice to shown a 5 or 6? *[(MTH1201.1,MTH1201.2)(Understand/LOCQ)]*

6 + 6 = 12

Group - C

4. (a) Find the real root of the equation $x^3 - x - 1 = 0$ using the Regula-Falsi method correct to 3 decimal places. *[(MTH1201.3)(Understand/LOCQ)]*

(b) Solve the following system by LU factorization method:
$3x_1 + 2x_2 - 4x_3 = 12$
$-x_1 + 5x_2 + 2x_3 = 1$
$2x_1 - 3x_2 + 4x_3 = -3$ *[(MTH1201.3)(Apply/IOCQ)]*

6 + 6 = 12

5. (a) Assuming the step length $h = 0.2$, use the Runge-Kutta formula of fourth order to find the numerical solution at $x = 0.8$ for the following initial value problem

$$\frac{dy}{dx} = x + y,\ y(0.4) = 0.41.$$ *[(MTH1201.3)(Apply/IOCQ)]*

(b) Solve the following system of equations by Gauss-Seidel method, correct to 3 decimal places:

$3x + 4y + 15z = 54.8$

$x + 12y + z = 39.66$

$10x + y - 2z = 7.74$ *[(MTH1201.3)(Evaluate/HOCQ)]*

6 + 6 = 12

Group - D

6. (a) Show that the number of vertices in a binary tree is always odd. *[(MTH1201.4)(Understand/LOCQ)]*

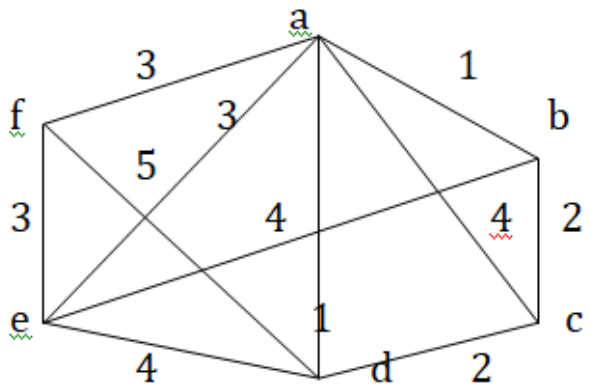
(b) Show that the number of pendant vertices in a binary tree is $\frac{n+1}{2}$, where n is the number of vertices in the tree. *[(MTH1201.4)(Remember/LOCQ)]*

(c) Draw the graph whose incident matrix is given below

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$ *[(MTH1201.4)(Understand/LOCQ)]*

3 + 4 + 5 = 12

7. (a) Use Prim's algorithm to find the minimal spanning tree and length of the minimal spanning tree for the following graph: *[(MTH1201.4)(Evaluate/HOCQ)]*



(b) If a simple regular graph has n vertices and 24 edges, find all possible values of n. *[(MTH1201.4)(Apply/IOCQ)]*

7 + 5 = 12

Group - E

8. (a) Evaluate $\mathcal{L}\left\{e^{-3t}\frac{Sin2\,t}{t}\right\}$. *[(MTH1201.5,MTH1201.6)(Evaluate/HOCQ)]*

(b) Find $\mathcal{L}^{-1}\left\{\frac{S^2}{(S^2+a^2)^2}\right\}$. *[(MTH1201.5,MTH1201.6)(Evaluate/HOCQ)]*

(c) Prove that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{sin\theta}} \times \int_0^{\frac{\pi}{2}} \sqrt{sin\theta} d\theta = \pi$. *[(MTH1201.5,MTH1201.6)(Understand/IOCQ)]*

4 + 4 + 4 = 12

9. (a) Find $\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2(s-2)}\right\}$ using convolution theorem.

[(MTH1201.5, MTH1201.6) (Evaluate/HOCQ)]

(b) Using Laplace Transform, find the solution of the following initial value problem $y''(t) + y(t) = 8\, cost$, where $y(0) = 1, y'(0) = -1$.

[(MTH1201.5, MTH1201.6)(Understand/LOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	43.75	21.87	34.38

Course Outcome (CO):

After the completion of the course students will be able to

MTH1201. 1. Demonstrate the knowledge of probabilistic approaches to solve wide range of engineering problem.
MTH1201. 2. Recognize probability distribution for discrete and continuous variables to quantify physical and engineering phenomenon.
MTH1201. 3. Develop numerical techniques to obtain approximate solutions to mathematical problems where analytical solutions are not possible to evaluate.
MTH1201. 4. Analyze certain physical problems that can be transformed in terms of graphs and trees and solving problems involving searching, sorting and such other algorithms.
MTH1201. 5. Apply techniques of Laplace Transform and its inverse in various advanced engineering problems.
MTH1201. 6. Interpret differential equations and reduce them to mere algebraic equations using Laplace Transform to solve easily.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*