

**PHYSICS - I**  
**(PHY 1001)**

Time Allotted : 2½ hrs

Full Marks : 60

*Figures out of the right margin indicate full marks.*

*Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.*

*Candidates are required to give answer in their own words as far as practicable.*

**Group - A**

1. Answer any twelve:

**12 × 1 = 12**

*Choose the correct alternative for the following*

- (i) For Young's double slit experiment using a monochromatic light source, the fringe width is inversely proportional to  
 (a) distance between the centres of two slits (b) wavelength of light  
 (c) distance of the screen from the slits (d) intensity of the light source
- (ii) In a weakly damped motion the curve representing the relaxation time and damping factor is a  
 (a) straight line (b) parabola  
 (c) hyperbola (d) semi-cubic parabola
- (iii) In a purely inductive circuit the current  
 (a) lags the voltage (b) leads the voltage  
 (c) is in phase with the voltage (d) can take up any phase with the voltage
- (iv) Which of the following represents a circularly polarized light vector?  
 (a)  $\vec{E}(z, t) = E_0 \cos(kz - \omega t)\hat{i} + E_0 \cos(kz - \omega t)\hat{j}$   
 (b)  $\vec{E}(z, t) = E_0 \cos(kz - \omega t)\hat{i} + E_0 \cos(kz - \omega t + \frac{\pi}{2})\hat{j}$   
 (c)  $\vec{E}(z, t) = E_0 \cos(kz - \omega t)\hat{i} + E_0 \cos(kz - \omega t + \frac{\pi}{3})\hat{j}$   
 (d)  $\vec{E}(z, t) = 2E_0 \cos(kz - \omega t)\hat{i} + E_0 \cos(kz - \omega t + \frac{\pi}{2})\hat{j}$
- (v) Which of the following is a pseudo-force?  
 (a) Gravitational force (b) Frictional force  
 (c) Electrostatic force (d) Coriolis force.
- (vi) For a particle confined to move in YZ plane under the influence of central force, for a given constant  $h$ , the angular momentum may have the form  
 (a)  $h\hat{k}$  (b)  $h(\hat{i} + \hat{j})$  (c)  $h(\hat{j} + \hat{k})$  (d)  $h\hat{i}$
- (vii) Which of the following is the eigen function of the operator  $\frac{d}{dx}$ ?  
 (a)  $\sin x$  (b)  $e^{-2x}$  (c)  $e^{x^2}$  (d)  $\cos x$ .

- (viii) The two coherent sources in Newton's ring experiment are obtained by  
 (a) division of amplitude (b) division of wave front  
 (c) division of frequency (d) division of wavelength
- (ix) The ground state energy for particle of mass  $m$  in a one dimensional box ( $0 < x < a$ ) is given by  
 (a) 0 (b)  $\frac{h^2}{8ma^2}$  (c)  $\frac{h^2}{8a^2}$  (d)  $\frac{h^2}{2ma^2}$
- (x) If  $[A, B] = 1$ , then the expression  $[A, B^2]$  is equal to  
 (a)  $BA$  (b)  $AB$  (c)  $2B$  (d)  $2A$

*Fill in the blanks with the correct word*

- (xi) The eigenvalues of a Hermitian operator are \_\_\_\_\_.
- (xii) The normalization constant  $N$  for the wavefunction of a particle  $\psi = N \sin\left(\frac{4\pi x}{L}\right)$  confined in the region  $0 \leq x \leq L$  is \_\_\_\_\_.
- (xiii) Two frames of reference are called \_\_\_\_\_ if the position vector of a particle in both the frames are related by  $\vec{r}' = \vec{r} - \vec{v}t$ , where  $\vec{v}$  is constant.
- (xiv) A rotating frame is an example of \_\_\_\_\_ frame.
- (xv) A half wave plate introduces a phase difference \_\_\_\_\_.

### Group - B

2. (a) The orbit of a planet rotating around the sun is given by  $r(\theta) = \frac{5}{3+2\cos\theta}$ .  
 (i) Evaluate the eccentricity of the orbit.  
 (ii) Evaluate the mean distance of the planet from the sun.  
 (iii) Conclude that the square of the time period is proportional to the cube of the mean distance of the planet from the sun. [[CO2](Evaluate/HOCQ)]
- (b) The position vector of a particle is given by  $\vec{r} = 3t^2\hat{i} + 5t\hat{k}$  relative to a rotating frame with angular velocity  $\vec{\omega} = 3\hat{i}$ . Find the Coriolis force on the particle. [[CO1](Remember/LOCQ)]  
**(3 + 3 + 3) + 3 = 12**
3. (a) Construct the operator relation  $\left(\frac{d}{dt}\right)_{fix} = \left(\frac{d}{dt}\right)_{rot} + \vec{\omega} \times$  where the symbols have their usual meanings. [[CO1](Apply/IOCQ)]
- (b) The trajectory of a particle is given by  $r(\theta) = Ae^{-b\theta}$ ,  $A$  and  $b$  are nonzero positive constant. Discover the force law and justify the force as a central force. [[CO5](Analyse/IOCQ)]
- (c) Examine the energy conservation principle for a particle moving under the force law  $\vec{F}(\vec{r}) = -\frac{K}{r^5}\hat{r}$ ,  $K$  being a constant. [[CO5](Analysing/IOCQ)]  
**4 + 4 + 4 = 12**

## Group - C

4. The steady-state displacement of a particle performing forced oscillation is given by  $x(t) = \frac{f}{\sqrt{D}} \cos(\omega t - \theta)$ , where  $D = (\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2$  and  $\omega_0, \omega, \gamma$  are natural frequency, forcing frequency and damping factor respectively.
- Find the velocity of the particle at  $t = \frac{\pi}{2\omega}$ .
  - Find the value of  $\omega$  for which the velocity amplitude is maximum.
  - Find the maximum value of the velocity amplitude.
  - Illustrate the velocity resonance curve for two different values of  $\gamma$ .
- [[CO1](Understand/LOCQ)]  
**(3 + 3 + 3 + 3) = 12**

5. (a) The equation of a weakly damped motion is given by  $5\ddot{x} + 0.02\dot{x} + 125x = 0$ . Find the relaxation time and logarithmic decrement. [[CO1](Remember/LOCQ)]
- (b) The equation of a damped motion is given by  $\ddot{x} + 10\dot{x} + 25x = 0$ .
- Identify the type of damping.
  - Develop the expression of displacement of a pendulum bob as a function of time given the initial conditions  $x(0) = 3 \text{ cm}, v(0) = -5 \text{ cm/s}$ . [[CO2](Apply/IOCQ)]
  - Show that the pendulum bob can never pass through the equilibrium point for any finite value of time. [[CO3](Understand/LOCQ)]
- (2 + 2) + (2 + 3 + 3) = 12**

## Group - D

6. (a) Explain that  $\psi(y, t) = B e^{-(2y+3t)^2}$  is a solution of a one-dimensional wave equation. Find the velocity of the wave. [[CO3](Evaluate/HOCQ)]
- (b) The light vector of a plane polarized light is given by  $\vec{E}(y, t) = 2 \cos(\omega t - ky)\hat{i} + 3 \cos(\omega t - ky)\hat{k}$ . Find the equation of its plane of vibration. [[CO3](Remember/LOCQ)]
- (c) The intensity of the superposition (resultant) of two light vectors represented by  $\vec{E}_1$  and  $\vec{E}_2$  is given by  $I = \langle E_1^2 \rangle + \langle E_2^2 \rangle + 2 \langle \vec{E}_1 \cdot \vec{E}_2 \rangle$  where  $\langle \vec{E}_1 \cdot \vec{E}_2 \rangle = \frac{1}{T} \int_0^T \vec{E}_1 \cdot \vec{E}_2 dt$ .
- Identify the interference term in the above expression for the resultant intensity. [[CO3](Identiy/IOCQ)]
  - Evaluate it for the two light vectors given as  $\vec{E}_1(z, t) = \cos(kz - \omega t)\hat{j}$  and  $\vec{E}_2(z, t) = \cos(kz - \omega t)\hat{j}$ . [[CO3](Evaluate/HOCQ)]
- (3 + 1) + 4 + (1 + 3) = 12**
7. (a) In the Newton's rings experimental setup, if the diameter of fourth dark ring goes from 2.52 mm to 2.21 mm before and after pouring a clear liquid in the air gap between the plano convex lens and concave lens, estimate the refractive index of the clear liquid. Also, evaluate the radius of curvature of the plano convex lens if it is illuminated by a monochromatic light of wavelength 6000 Å. [[CO3](Evaluate/HOCQ)]

- (b) Create an expression for a plane polarized disturbance of angular frequency  $\omega$  propagating in the positive  $y$  – direction such that the  $\vec{E}$  field makes an angle of  $30^\circ$  with the positive  $x$  – direction. [[CO3](Create/HOCQ)]
- (c) (i) For obtaining a double slit diffraction pattern, the individual slits of thickness 0.15mm are used. The distance between the centres of each slit is 0.45mm. Find the missing orders for a double slit Fraunhofer pattern.
- (ii) What is an unpolarised light? How can one obtain a circularly polarised light from an unpolarised light? [[CO3](Remember/LOCQ)]

$$(2 + 2) + 4 + [2 + (1 + 1)] = 12$$

### Group - E

8. (a) The quantum state of a system is given by  $\varphi = \frac{1}{\sqrt{6}}\varphi_1 + \sqrt{\frac{5}{6}}\varphi_2$  where,  $\varphi_1$  and  $\varphi_2$  are normalized eigenstates of a Hermitian operator. Find the probability of finding the system in the respective states. [[CO5](Understand/LOCQ)]
- (b) Deduce that the eigenstates of a Hermitian operator corresponding to two different eigenvalues are orthogonal. [[CO5](Evaluate/HOCQ)]
- (c) Demonstrate that  $-i\frac{d}{dx}$  is a Hermitian operator relative to the definition of inner-product for two arbitrary states  $\varphi, \psi$  as  $(\varphi, \psi) = \int_{-\infty}^{\infty} \varphi^* \psi dx$  where  $\varphi(\pm\infty) = \psi(\pm\infty) = 0$ . [[CO6](Understanding/LOCQ)]
- $$4 + 4 + 4 = 12$$
9. (a) Discuss that if for two observables  $A$  and  $B$   $[A, B] = AB - BA = 0$  and  $\psi$  is the only eigenvector of  $A$  corresponding to the eigenvalue  $a$ ,  $B\psi$  is also an eigenvector of  $A$  corresponding to the same eigenvalue. [[CO5](Create/HOCQ)]
- (b) A single particle is confined in a one dimensional box  $0 < x < a$ .
- (i) Find the energy difference between 1<sup>st</sup> and 2<sup>nd</sup> excited states and plot the corresponding wave functions as a function of  $x$ .
- (ii) Find the uncertainty in the measurement of momentum for 1<sup>st</sup> excited state. [[CO6](Remember/LOCQ)]
- $$4 + (4 + 4) = 12$$

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	47.9	21.9	30.2

#### Course Outcome (CO):

After the completion of the course students will be able to

1. Understanding physical systems in terms of their modelling of time evolution.
2. Comprehending wave interpretation of natural phenomena and implications of allied observations.
3. Understanding theoretical backgrounds associated to some experiments based on wave phenomena.
4. Grasping an analytic view of micro and macroscopic world.
5. Accessing the knowledge of the behaviour of a particle under the influence of different potential.
6. Understanding conservative systems based on their particle and wave nature.

\*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.