B.Tech/ AEIE/BT/CE/CHE/CSE/ECE/EE/IT/ME/ 1st Sem/MATH-1101/2016

2016

MATHEMATICS 1

(MATH 1101)

Time Allotted : 3 hrs ***Full Marks : 70***

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

GROUP - A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **[10 x 1=10]**

i) The rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is

a) 3 b) 1 c) 2 d) 0

ii) If *A* be an orthogonal matrix then A^{-1} is

a) Symmetric b) Skew- symmetric c) Orthogonal d) Idempotent

iii) Every scalar matrix is

a) diagonal b) symmetric c) skew-symmetric d) orthogonal

iv) In M.V.T $f(h) = f(0) + h f'(\theta h)$, $0 < \theta < 1$, if $f(x) = \frac{1}{1+x}$ and h = 3; then value of θ is:

a) 1 b) $\frac{1}{3}$ c) $\frac{1}{\sqrt{2}}$ d) none of these

v) The series $\sum \frac{2^n}{e^n}$ is

a) oscillatory b) divergent c) convergent d) nothing can be said

vi) Which of the following does not satisfy Rolle's Theorem in [-2, 2] ?

a) x^2 b) $\frac{1}{x-1}$ c) x d) none of these

vii) The sequence $\left\{\frac{n}{1+n^2}\right\}$ is

a) convergent b) divergent c) oscillatory d) none

viii) If $f(x,y) = \frac{x}{y} + \frac{y}{x}$ then $xf_x + yf_y =$

a) 0 b) -1 c) 2 d) $f(x,y)$

ix) The series $\sum \frac{1}{n^p}$ is convergent if

a) p < 1 b) p = 0 c) p > 1 d) p = 1

x) The value of $\int_0^{\pi/2} \sin^5 x \cos^6 x \, dx$ is

a) $\frac{2}{693}$ b) $\frac{8}{693}$ c) $\frac{4}{693}$ d) $\frac{8\pi}{693}$

GROUP - B

2 a) Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

b) Prove that orthogonal matrices are non-singular.

c) Find the rank of the following matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$$

6+2+4=12

3 a) Verify Cayley Hamilton theorem for $\begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$.

Hence find A^{-1}

b) Solve by matrix method, the equations

$x + y + z = 8$

$x - y + 2z = 6$

$3x + 5y - 7z = 14$

6+6=12

GROUP - C

4 a) If $x_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots\dots\dots\dots + \frac{1}{n.(n+1)}$ then show that $\{x_n\}$ is a bounded monotonic increasing sequence.

b) Verify the Rolle's theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ in [1,3]

c) Prove that the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$ is absolutely convergent, when $|x| < 1$ and conditionally convergent when $|x| = 1$

4+3+5=12

5 a) Using Lagrange's Mean Value Theorem prove that

$$\frac{2x}{1-x^2} > \log\left(\frac{1+x}{1-x}\right) > 2x, \quad 0 < x < 1$$

b) For what values of x, the following series is convergent

$$\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \cdots$$

6+6=12

GROUP - D

6 a) if $u = \cos^{-1}\left\{\frac{x+y}{\sqrt{x}+\sqrt{y}}\right\}$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$

b) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$, find $\frac{\partial(u,v)}{\partial(x,y)}$

6+6=12

7 a) Find the maxima and minima of the function

$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

b) if $y = e^{ax} \cos bx$, then show that

$y_n = (a^2 + b^2)^{n/2} e^{ax} \sin(bx + n \tan^{-1} b/a)$ where a and b are non-zero constants.

6+6=12

GROUP - E

8 a) Find the maximum value of the directional derivative of $\phi = x^2+y^2+z^2$ at the point (1,2,3). Find also the direction in which it occurs.

b) Evaluate $\iint_D (4xy - y^3)\, dx\, dy$, D is the region bounded by $y = \sqrt{x}$, $y = x^3$

6+6=12

9 a) Verify Green's theorem in the plane for $\oint_c [(xy + y^2)dx + x^2\, dy]$ where C is the closed curve of the region bounded by y = x and $y = x^2$

b) Use Divergence theorem to evaluate $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F} = xy\,\hat{i} - \frac{y^2}{2}\,\hat{j} + z\hat{k}$ and the surface consists of the three surfaces $z = 4 - 3x^2 - 3y^2$ $1 \le z \le 4$ on the top, $x^2 + y^2 = 1$, $0 \le z \le 1$ on the sides and z = 0 at the bottom.

6+6=12