ADVANCED DISCRETE MATHEMATICS AND STATISTICAL METHODS
(MATH 5101)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The mean and variance of a Binomial distribution $B(n,p)$ are 4 and 2 respectively. Then the values of n and p are
(a) $(8,\frac{1}{2})$ (b) $(4,\frac{1}{2})$ (c) $(8,\frac{1}{4})$ (d) (4, 2).

(ii) If the two bi-variates (x,y) and (u,v) are such that $x=-2u+4$ and $y=3u-6$ then
(a) $r_{xy}=r_{uv}$ (b) $r_{xy}=-6r_{uv}$
(c) $r_{xy}=6r_{uv}$ (d) $r_{xy}=-r_{uv}$.

(iii) The mode and median of the observations 4, 6, 6, 8, 3, 8, 8 and 4 are
(a) 8 and 6 (b) 8.5 and 6.5 (c) 5 and 7 (d) 4 and 3.

(iv) An urn contains 3 white and 5 black balls. Two balls are drawn one by one without replacement. The probability that both the balls are black is
(a) 5/16 (b) 5/8 (c) 3/8 (d) 1/2.

(v) $C(n,0)+C(n,1)+C(n,2)+C(n,3)+\cdots+C(n,n)=$
(a) 2^{n-1} (b) 3^n (c) 2^n (d) 3^n-1

(vi) The generating function for the sequence {0, 0, 0, 1, 1, 1, 1, 1, 1, … } is
(a) $\frac{x}{1-x^2}$ (b) $\frac{x^3}{1-x}$ (c) $\frac{1}{1-x^2}$ (d) $\frac{1}{1-x^3}$.

(vii) The number of 3-digit numbers that can be formed using the digits 1, 3, 4, 5, 6, 8 and 9 is
(a) 49 (b) 343 (c) 21 (d) 7

(viii) The probability mass function of a discrete random variable X is defined as $P(X=1)=2k^2$, $P(X=2)=5k-7k^2$, $P(X=4)=2k-1$, and $P(X=x)=0$ for $x\neq 1,2,4$, then the value of k is
(a) 2/5 (b) 1/5 (c) 1 (d) 1/2.

(ix) An urn contains 3 white and 4 red balls. Two balls are drawn one by one without replacement. The probability that the first ball is white and the second ball is red is

(a) 3/7 (b) 2/7 (c) 4/7 (d) 1/7.

(x) The graph K_4 is

(a) Euler but not Hamiltonian graph
(b) Hamiltonian but not Euler graph
(c) both Euler and Hamiltonian graph
(d) neither Euler nor Hamiltonian graph.

Fill in the blanks with the correct word

(xi) If A and B are independent events then the conditional probability $P(A/B) =$ _____.

(xii) If $a_n = a_{n-1} + 9, n \geq 1$, and $a_0 = 5$ then $a_n =$ _____.

(xiii) The chromatic polynomial of a null graph with 3 vertices is _____.

(xiv) The number of ways in which 7 women and 3 men can be arranged in a row if the 3 men always stand next to each other is _____.

(xv) If X is normally distributed with zero mean and unit variance, then $Var(-3X + 4$ is _____.

Group - B

2. (a) The chance that a doctor will diagnose a certain disease correctly is 60%. The chance that a patient will die by his treatment after the correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of the doctor who had the disease dies. What is the probability that the disease was diagnosed correctly. *[(MATH5101.1, MATH5101.2)(Apply/IOCQ)]*

(b) Show that $f(x) = \begin{cases} x, & 0 < x \leq 1 \\ k - x, & 1 < x \leq 2 \\ 0, & otherwise \end{cases}$

is a density function for a suitable value of k. Find the distribution function $F(x)$ of random variable X. *[(MATH5101.1, MATH5101.2)(Evaluate/HOCQ)]*

6 + 6 = 12

3. (a) Two persons **A** and **B** throw alternatively with a pair of dice. **A** wins if he throws 6 before **B** throws 7 and **B** wins if he throws 7 before **A** throws 6. If **A** begins, find his probability of winning. *[(MATH5101.1, MATH5101.2)(Understand/LOCQ)]*

(b) Two newspapers, X and Y are published in a certain city. It is estimated from a survey that 16% read X, 14% read Y and 5% read both the newspapers. Find the probabilities that a randomly selected person (i) does not read any newspaper (ii) reads only Y. *[(MATH5101.1, MATH5101.2)(Understand/LOCQ)]*

6 + 6 = 12

Group - C

4. (a) The length of bolts produced by a machine is normally distributed with mean 4 and standard deviation 0.5. A bolt is defective if its length does not lie in the interval (3.8, 4.3). Find the percentage of defective bolts produced by the machine. *[(MATH5101.1, MATH5101.2)(Apply/IOCQ)]*

(b) A defective die is thrown ten times independently. The probability that an even number will appear 5 times is twice the probability that an even number will appear 4 times. What is the probability that odd face appear in each of the ten throws. *[(MATH5101.1, MATH5101.2)(Analyze/IOCQ)]*

6 + 6 = 12

5. (a) The following data gives the hardness (x) and tensile strength (y) for some specimens of a material, in certain units. Find the correlation coefficient and the regression line of x on y.

x	23.3	17.5	17.8	20.7	18.1	20.9	22.9
y	4.2	3.8	4.6	3.2	5.2	4.7	4.4

[(MATH5101.1, MATH5101.2) (Evaluate/HOCQ)]

(b) Calculate the median and mode of the following frequency distribution:

Marks:	10-19	20-29	30-39	40-49	50-59	60-69
Frequency:	8	11	15	17	17	7

[(MATH5101.1, MATH5101.2)(Apply/IOCQ)]

6 + 6 = 12

Group - D

6. (a) (i) Solve the recurrence relation $a_n - 7a_{n-1} + 12a_{n-2} = 0$ for $n \geq 2$ by the method of characteristic roots, where $a_0 = 2, a_1 = 5$. *[(MATH5101.1, MATH5101.2, MATH5101.3)(Apply/IOCQ)]*

(ii) Write the general solution a_n of the linear homogeneous recurrence relation whose characteristic polynomial is $(t^2 - 5t + 6)^2(t - 3)$. *[(MATH5101.1, MATH5101.2, MATH5101.3)(Remember/LOCQ)]*

(b) How many bit strings of length 10 contain (i) exactly four 1's; (ii) at most four 1's; (iii) an equal numbers of 0's and 1's. *[(MATH5101.1, MATH5101.2, MATH5101.3)(Analyse/IOCQ)]*

(3 + 3) + 6 = 12

7. (a) There are 250 students in an engineering college. Of these, 188 have taken a course in FORTRAN, 100 have taken a course in C and 35 have taken a course in JAVA. Further 88 have taken courses in both FORTRAN and C, 23 have taken courses in both C and JAVA and 29 have taken courses in both FORTRAN and JAVA. If 19 of these students have taken all the three courses, how many of these 250 students have not taken a course in any of these programming languages? *[(MATH5101.1, MATH5101.2, MATH5101.3) (Remember/LOCQ)]*

(b) Prove Newton's identity: $C(n,r)C(r,k) = C(n,k)C(n-k,r-k)$ for integers $n \geq r \geq k \geq 0$. *[(MATH5101.1, MATH5101.2, MATH5101.3)(Apply/IOCQ)]*

6 + 6 = 12

Group – E

8. (a) Let G be a planar graph with 10 vertices, 3 components and 9 edges. Find the number of regions in G. *[(MATH5101.1, MATH5101.2, MATH5101.4)(Remember/LOCQ)]*

   (b) Draw Kuratowski's first graph and further prove that it is non-planar. *[(MATH5101.1, MATH5101.2, MATH5101.4) (Remember/LOCQ)]*

   (c) Find whether the polynomials
   (i) $x^4 - 5x^3 + 6x^2$
   (ii) $x^3 + 6x^2 - 6x - 1$
   are possible chromatic polynomial of any graph. Justify your answer.
   [(MATH5101.1, MATH5101.2, MATH5101.4) (Analyse/IOCQ)]

4 + 4 + (2 + 2) = 12

9. (a) Find the chromatic polynomial of a disconnected graph with three components, where the components are C_3 (cycle graph with three vertices), K_5 (complete graph with 5 vertices) and a null graph with 3 vertices. Hence find the chromatic number. *[(MATH5101.1, MATH5101.2, MATH5101.4)(Remember/LOCQ)]*

   (b) Define self dual of a graph. Draw the dual of K_4 (complete graph with 4 vertices). Is it a self dual graph? Justify your answer.
   [(MATH5101.1, MATH5101.2, MATH5101.4)(Analyse/IOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	36.46	51.04	12.5

Course Outcome (CO):

After the completion of the course students will be able to

MATH5101.1 To understand the mathematical fundamentals that is prerequisites for a variety of courses like Data mining, Network protocols, analysis of Web traffic, Computer security, Software engineering, Computer architecture, operating systems, distributed systems, Bioinformatics, Machine learning.

MATH5101.2 To develop the understanding of the mathematical and logical basis to many modern techniques in information technology like machine learning, programming language design, and concurrency.

MATH5101.3 To study the principles of enumeration.

MATH5101.4 To equip oneself with the techniques used in graph theory.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*