COMPUTATIONAL METHODS IN ENGINEERING
(MECH 4128)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) A mathematical model of a physical system or process is
(a) formulation of equations (b) description of the system
(c) experimentation with the system (d) explanation of the result.

(ii) Which of the following is an iterative method?
(a) Gauss Seidel method (b) Gauss Jordan method
(c) Gauss elimination method (d) Factorization method.

(iii) The number of significant digits in 2.0108 is
(a) 3 (b) 5 (c) 6 (d) 7.

(iv) The first-order polynomial in the finite difference linear fit scheme is
(a) $P_1(x) = \frac{(x-x_{i+1})}{(x_i-x_{i+1})} f_{i+1} - \frac{(x-x_i)}{(x_{i+1}-x_i)} f_i$ (b) $P_1(x) = \frac{(x-x_{i+1})}{(x_i-x_{i+1})} f_i - \frac{(x-x_i)}{(x_{i+1}-x_i)} f_{i+1}$
(c) $P_1(x) = \frac{(x-x_{i+1})}{(x_i-x_{i+1})} f_i + \frac{(x-x_i)}{(x_{i+1}-x_i)} f_{i+1}$ (d) $P_1(x) = \frac{(x-x_{i+1})}{(x_i-x_{i+1})} f_{i+1} - \frac{(x-x_i)}{(x_{i+1}-x_i)} f_{i-1}$

(v) Consider the linear and homogeneous second-order PDE $A\frac{\partial^2\varphi}{\partial x^2} + B\frac{\partial^2\varphi}{\partial x\,\partial y} + C\frac{\partial^2\varphi}{\partial y^2} + D\frac{\partial\varphi}{\partial x} + E\frac{\partial\varphi}{\partial y} + F\varphi + G = 0$. The PDE is classified as parabolic in characteristics if
(a) $B^2 - 4AC > 0$ (b) $B^2 - 4AC < 0$
(c) $B^2 - 4AC = 0$ (d) $B^2 - 4AC \geq 0$

(vi) Which of the following is also known as the Newton-Raphson method?
(a) Chord method (b) Tangent method
(c) Diameter method (d) Secant method.

(vii) In the LU decomposition method of solution, the coefficient matrix is decomposed into
(a) Lower and upper triangular matrix both
(b) Lower triangular matrix only
(c) Upper triangular matrix only
(d) Either lower or upper triangular matrix, depending on its rank.

(viii) Which one among the following is not a curve fitting process?
(a) Linear regression (b) Multiple linear regression
(c) Newton's interpolating polynomial (d) Simpson's $1/3^{rd}$ rule.

(ix) Which of the following equations is parabolic?
(a) $f_{xy} - f_x = 0$ (b) $f_{xx} + 2f_{xy} + f_{yy} = 0$
(c) $f_{xx} + 2f_{xy} + 4f_{yy} = 0$ (d) None of the above equations.

(x) Match the items in columns I and II:

Column I	Column II
P. Gauss-Seidel method	1. Interpolation
Q. Forward Newton-Gauss method	2. Non-linear differential equations
R. Runge-Kutta method	3. Numerical integration
S. Trapezoidal Rule	4. Linear algebraic equations

(a) P-1, Q-4, R-3, S-2 (b) P-1, Q-4, R-2, S-3
(c) P-1, Q-3, R-2, S-4 (d) P-4, Q-1, R-2, S-3.

Fill in the blanks with the correct word

(xi) The finite difference form of $\partial^2 u/\partial x^2$ is _____.

(xii) Using the trapezoidal rule with step size 1, the value of the definite integral $\int_1^3 \frac{dx}{x}$ is _____.

(xiii) A system of equations contains a real parameter a:
$2x + 3y = 5$
$4x + ay = 8$
The value of a for which the system has no solution is _____.

(xiv) The linear PDE $u_{xx} + u_{yy} + 3u_{xy} = 0$ of second order is classified as _____

(xv) The linear PDE $x^2 u_{xx} + (1 - y^2)u_{yy} = 0$ $(-\infty < x < \infty,\ -1 < y < 1)$ of second order is _____ (Classify).

Group - B

2. (a) Find a root of the equation $x^3 - x - 11 = 0$ using the Bisection method correct to two decimal places. [(CO2)(Apply/IOCQ)]
(b) Find a real root of the equation $x^3 + x - 1 = 0$ by the Regula-falsi method correct to two decimal places. *[(CO2)(Apply/IOCQ)]*

6 + 6 = 12

3. (a) Find a root of the equation $x^3 - 2x - 5 = 0$ correct to two decimal places, using the Newton-Raphson method. *[(CO2)(Apply/IOCQ)]*
(b) Find a root of the equation $x^3 - 3x + 1 = 0$ correct to two decimal places, using the Newton-Raphson method. *[(CO2)(Analyze/IOCQ)]*

6 + 6 = 12

Group - C

4. (a) Apply the Gauss elimination method to solve the equations $2x + 2y + z = 12;\ 3x + 2y + 2z = 8;\ 5x + 10y - 8z = 10.$ *[(CO2)(Apply/IOCQ)]*

   (b) Fit a polynomial of degree three which takes the following values:

x:	3	5	7	9
y:	6	24	60	120

   [(CO3)(Analyze/IOCQ)]

   6 + 6 = 12

5. (a) Apply the Gauss-Seidel iteration method to solve the system of equations $2x + y + 6z = 9;\ 8x + 3y + 2z = 13;\ x + 5y + z = 7$ upto third iteration.

   [(CO2) (Apply/IOCQ)]

   (b) Use Lagrange's interpolation formula to fit a polynomial to the following data:

x:	1	2	3	4
y:	12	10	15	18

   Hence find y (1.5), y (2.5) and y (3.5). *[(CO3)(Analyze/IOCQ)]*

   6 + 6 = 12

Group - D

6. (a) Evaluate the definite integral $\int_0^6 \frac{dx}{(1+x^2)}$ by using (i) Trapezoidal rule and (ii) Simpson's 3/8 rule. Divide the interval (0, 6) into four parts each of width h = 1.

   [(CO4) (Apply/IOCQ)]

   (b) Using Euler's method, find an approximate value of y corresponding to x = 0.1, given that $dy/dx = (y - x/y + x)$ and $y(x = 0) = 1$. *[(CO4)(Analyze/IOCQ)]*

   6 + 6 = 12

7. (a) Using modified Euler's method, find the approximate values of y (0.2) and y (0.4), given that$dy/dx = y + e^x$ and $y(x = 0) = 0$. *[(CO5)(Apply/IOCQ)]*

   (b) Using the Picard's method of successive approximation, obtain a solution upto the third approximation of the equation $dy/dx = xy$ such that $y(x = 0) = 1$.

   [(CO5)(Apply/IOCQ)]

   6 + 6 = 12

Group - E

8. (a) Using the Taylor series method, obtain the values of y at x = 0.1 and at x=0.2 by solving the equation $dy/dx = (x^2y - 2)$ such that $y\ (x = 0) = 1$. *[(CO5)(Apply/IOCQ)]*

   (b) Apply the Runge-Kutta fourth-order method to find an approximate value of y when x = 0.2, given that $dy/dx = 2(x + y)$ and $y\ (x = 0) = 1$. *[(CO5)(Analyze/IOCQ)]*

   7 + 5 = 12

9. (a) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the square mesh of Fig. 1 with boundary values as shown. Consider uniform step-size. Carry out computations till the 5^{th} iteration level.

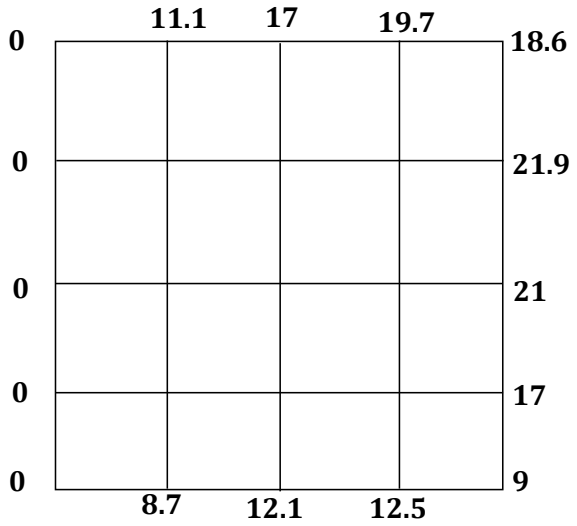

Fig. 1: Square mesh

[(CO6)(Investigate/HOCQ)]

(b) What is the classification of the linear partial differential equation $u_{xx} + 2u_{xt} + u_{tt} = 0$? Justify. *[(CO6)(Apply/IOCQ)]*

10 + 2 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	0	89.58	10.42

Course Outcome (CO):

After the completion of the course students will be able to

CO 1: Apply different mathematical models to obtain numerical solutions and classify different types of error.
CO 2: Analyze and solve a system of linear algebraic equations by different methods and find out the roots.
CO 3: Implement the regression and interpolation methods for curve fitting and apply different types of optimization techniques to solution of problems.
CO 4: Use different numerical integration methods for practical problems.
CO 5: Classify Initial-value and Boundary-value problems in order to formulate their solutions, implement different methods for their solutions, and solve Eigenvalue problems applied to physical systems.
CO 6: Classify linear, second-order partial differential equations (PDEs) as elliptic, parabolic, or hyperbolic, and apply the Finite Difference Method to formulate the solutions of different classes of PDEs.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*