(vii) Find the rank of the word LATE when the letters are arranged as in dictionary
(a) 13 (b) 14 (c) 15 (d) 16.

(viii) How many ways can 6 boys form a ring ?
(a) 24 (b) 120 (c) 96 (d) 720.

(ix) If the function f: R→R is defined as $f(x) = \begin{cases} 3x-4, & x>0 \\ -3x+2, & x\leq 0 \end{cases}$

Then $f^{-1}(2)$ is equal to
(a) {2} (b) {0, 2} (c) {2, -2} (d) none of these.

(x) The generating function of the sequence {0, 1, 0, -1, 0, 1, 0, -1, 0, ….} is
(a) $\frac{1}{1+x^2}$ (b) $\frac{x}{1+x^2}$ (c) $\frac{x^2}{1+x^2}$ (d) none of these.

Group - B

2. (a) (i) Prove that in the set of natural numbers, the relation 'x is multiple of y' is an equivalence relation.
(ii) Show that $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

(b) Show that the mapping is f: $\mathbb{N} \times \mathbb{N}$ defined by $f(n) = n - (-1)^n$, $n \in \mathbb{N}$ is bijective.

(4 + 3) + 5 = 12

3. (a) If $A \Delta B = A \Delta C$, then prove that B=C (where A, B, C are any three nonempty sets).

(b) Show that, the set of vectors {(1, 2, 2), (2, 1, 2), (2, 2, 1)} is linearly independent in R^3.

(c) Let L be the set of all lines in 3D space and R be a binary relationship on L, such that, two lines l1 and l2 are related, iff l1 lies on the plane containing l2. Test whether the relationship is Reflexive, Symmetric, Transitive.

3 + 3 + 6 = 12

Group - C

4. (a) i) Find the coefficient of x^5 in $(x^2 + x^3 + x^4 + \ldots)^4$.
ii) Solve the recurrence relation: $a_n - 7a_{n-2} + 6a_{n-3} = 0$ with initial conditions, $a_0 = 8$, $a_1 = 6$ and $a_2 = 22$ using generating function.

(b) Out of 5 males and 6 females, a committee of 5 is to be formed. Find the number of ways in which it can be formed, so that among the persons chosen in the committee there are
i) Exactly 3 males and 2 females.
ii) At least 2 males and 1 female.

(3 + 4) + 5 = 12

5. (a) Prove that, if any 30 people are selected, we may choose a subset of 5, so that all of them were born on the same day of the week.

(b) Find the Generating function for the numeric function $0<n+n^2>\infty$.

(c) Using generating function solve the recurrence relation
$a_n - 7a_{n-1} + 10a_{n-2} = 0$ for all $n > 1$ and $a_0 = 3$ and $a_1 = 3$.

3 + 4 + 5 = 12

Group – D

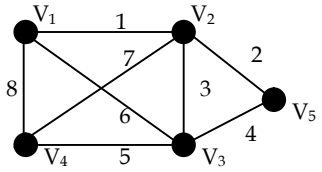
6. (a) Prove that a tree with n number of vertices has (n - 1) number of edges.

(b) Let G be a simple connected Planar graph with n vertices, e edges and f regions then prove that (i) $e \geq 3f / 2$ (ii) $e \leq 3n - 6$.

4 + (4 + 4) = 12

7. (a) Prove that a simple graph with n vertices and k components can have at most $(n - k)(n - k + 1) / 2$ edges.

(b) How many internal vertices does a full binary tree with h levels has? Apply Kruskal's algorithm to find the minimal spanning tree of the following weighted graph.



5 + (2 + 5) = 12

Group - E

8. (a) Define grammar of a language and the types. Give an example of a grammar which is Type 2 but not Type 3.

(b) Construct a grammar generating the following language:
$L = \{ \omega \in \{a, b\}^* : \omega \text{ is a palindrome} \}$

(2 + 3) + 7 = 12

9. (a) i) Define Mealy machine and Moore Machine.
ii) Construct a Moore Machine from the following Mealy machine:

Present State	Next State			
	a = 0		a = 1	
	State	Output	State	Output
S_0	S_0	1	S_1	0
S_1	S_3	1	S_3	1
S_2	S_1	1	S_2	1
S_3	S_2	0	S_0	1

(b) Consider the grammar G with V = {S, A, B}, $\sum$ = {a, b}, and P = {S → AB, S → bA, A → a, B → ba}. Find L (G).

(2 + 5) + 5 = 12

DISCRETE MATHEMATICS
(MCAP 1104)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A relationship set is called partially ordered set if it is
(a) reflexive, symmetric and transitive
(b) symmetric, transitive and antisymmetric
(c) reflexive, transitive and antisymmetric
(d) reflexive and symmetric and antisymmetric.

(ii) The coefficient of x^5 in the expansion of $(x^3+x^4+x^5+\ldots\ldots\ldots)^5$ is
(a) C(9,5) (b) C(5,9) (c) C(5,5) (d) C(9,9).

(iii) What is the maximum number of edges in a Graph G with n vertices?
(a) n(n+1)/2 (b) n(n-1)/2
(c) $n^2/2$ (d) $(1+n+n^3)/3$.

(iv) {a,b} ≤ V_T and S ∈ V_N, then S → ab is a
(a) type-0 grammar (b) type-1 grammar
(c) type-2 grammar (d) type-3 grammar.

(v) Six boys and four girls can sit in a row in
(a) 6! × 4! ways (b) 2 × 6! × 4! ways
(c) 2^{24} ways (d) none of these.

(vi) Let A={1, 2, 3, 4,..., 8, 9}, B={2, 4, 6, 8}, C={1, 3, 5, 7, 9}, D={3, 4, 5} and E={3, 5}. Then which set can equal X, if we are given the following information?
(a) X and B are disjoint (b) $X \subseteq A$ but $X \not\subset C$
(c) $X \subseteq D$ but $X \not\subset B$ (d) $X \subseteq C$ but $X \not\subset A$.