(vii) Which of the following does not always guarantee the convergence?
(a) Secant method (b) Regular falsi method
(c) Bisection method (d) Newton raphson method.

(viii) The degree of precision of Simpson's 1/3 rule is
(a) 0 (b) 2 (c) 1 (d) 3.

(ix) The power of a test is defined as
(a) 1 – P(Type I Error)
(b) 1 – P(Type II Error)
(c) P(Type I Error) P(Type II Error)
(d) 1 – P(Type I Error) P(Type II Error).

(x) The relation between shift operator 'E' and forward difference operator 'Δ' is given by
(a) Δ = 1 - E (b) E = 1 + Δ (c) E = Δ (d) E = Δ + 2.

Group - B

2. (a) For any two events A_1 and A_2, prove that
(i) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$
(ii) $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$
(iii) $P(A_1 \cap A_2) \geq P(A_1) + P(A_2) - 1$

(b) Find the probability that in the throw of two unbiased dice the sum of points will be even or less than 5.

(5 + 1 + 1) + 5 = 12

3. (a) A random variable X has the following probability density function:

$$f(x) = \begin{cases} k, & -2 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Determine the constant k.
(ii) What is the value of $P(|X|) > 1$?

(b) If the weekly wage of 10,000 workers in a factory follows normal distribution with mean and standard deviation Rs.70 and Rs.5 respectively, find the expected number of workers whose weekly wages are
(i) Between Rs.66 and Rs.72.
(ii) Less than Rs.66.
(iii) More than Rs.72.

Given that, $\int_0^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ = 0.1554 and 0.2881, according as z = 0.4 and 0.8.

(2 + 3) + (3 + 2 + 2) = 12

Group - C

4. (a) If $x_1, x_2, \ldots, x_n$ is a random sample from an infinite population with variance σ^2, and $\bar{x}$ is the sample mean, show that $\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is a biased estimator of σ^2.

(b) If the sample observations are 2, 4, 6, 8 and 10 from an infinite population with variance σ^2, determine an unbiased estimate of σ^2.

7 + 5 = 12

5. (a) Distinguish between the following, in context of hypothesis testing: Type I error and Type III error, one-sided test and two-sided test.

(b) In order to test whether a coin is unbiased, the coin is tossed 5 times and the null hypothesis of perfectness of the coin is rejected if and only if more than 4 heads are obtained. (i) What is the probability of Type I error? (ii) Find the probability of Type II error, when the corresponding probability of head is 0.2.

(4 + 2) + (3 + 3) = 12

Group – D

6. (a) What is relative error and percentage error?

(b) Given:

x	1	2	3	4	5	6	7	8
y	1	8	27	64	125	216	343	512

Construct the difference table and compute i) *f(1.5)* ii) *f(7.5)*.

2 + 10 = 12

7. (a) Establish Newton's forward interpolation formula.

(b) Evaluate $\int_0^1 \sqrt{(1-x^3)}\,dx$ using Simpson's 1/3rd rule dividing the range into 10 intervals.

(c) What is the geometrical significance of Trapezoidal method?

4 + 4 + 4 = 12

Group - E

8. (a) Evaluate y(0.1) using Euler's method $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 taking h = 0.02.

   (b) Find the real root of $x^3 - x - 11 = 0$ using bisection method.

   (c) Evaluate y(0.2) using Runge-kutta method of fourth order to calculate $\frac{dy}{dx} = y + x$, y(0) = 1 taking h = 0.1.

5 + 3 + 4 = 12

9. (a) State and explain Regular-Falsi method.

   (b) Solve the following system of equations using the Gauss Seidel method.
   4x+3y-5z = 2, x+y-z = 1, 8x+y–5z =4.

   (c) Find $\sqrt{45}$ using Newton-Raphson method.

4 + 5 + 3 = 12

NUMERICAL & STATISTICAL TECHNIQUES
(MCAP 1103)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

   (i) The chance that a leap year selected at random will contain 53 Wednesdays is
   (a) $^2/_7$ (b) 0 (c) 1 (d) $^6/_7$.

   (ii) If $P(\bar{A} \cup \bar{B}) = {}^5/_6$, $P(A) = {}^1/_2$ and $P(\bar{B}) = {}^2/_3$ then the events A and B are
   (a) mutually exclusive (b) mutually independent
   (c) mutually exhaustive (d) none of the above.

   (iii) For what value of a is $f(x) = a({}^1/_2)^x$, x = 0, 1, 2,……, a probability mass function of a random variable X?
   (a) $^1/_4$ (b) $^2/_7$ (c) $^1/_2$ (d) 1.

   (iv) If T_1 and T_2 are statistics with $E(T_1) = 2\theta_1 + 3\theta_2$ and $E(T_2) = \theta_1 + \theta_2$, then the unbiased estimator of θ_1 is
   (a) $3T_2 - T_1$ (b) $3T_2 + 2T_1$
   (c) $2T_2 - 3T_1$ (d) $3T_2T_1$.

   (v) The mean and standard deviation of a binomial distribution are 4 and $\sqrt{{}^8/_3}$ respectively. The values of the parameters n and p are
   (a) 11, $^3/_4$ (b) 12, $^2/_7$ (c) 12, $^1/_3$ (d) 11, $^4/_3$.

   (vi) If T is an unbiased estimator of θ, then T^2 a biased estimator of
   (a) θ^2 (b) $\sqrt{\theta}$ (c) θ (d) All of the above.